Unexpected connections

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These slides: www.maths.ed.ac.uk/~tl/perth

Keep yourself open

and don't neglect your larger self

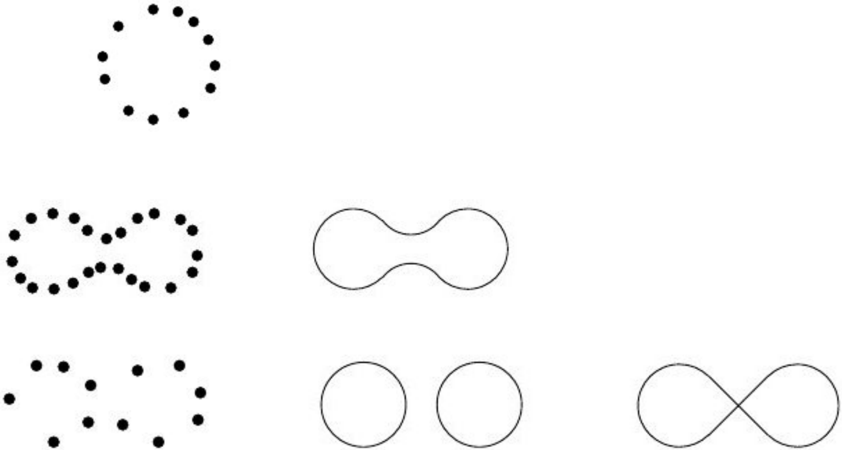
Applied maths

$\neq$ *applied differential equations*

$\neq$ *differential equations applied to physical problems*

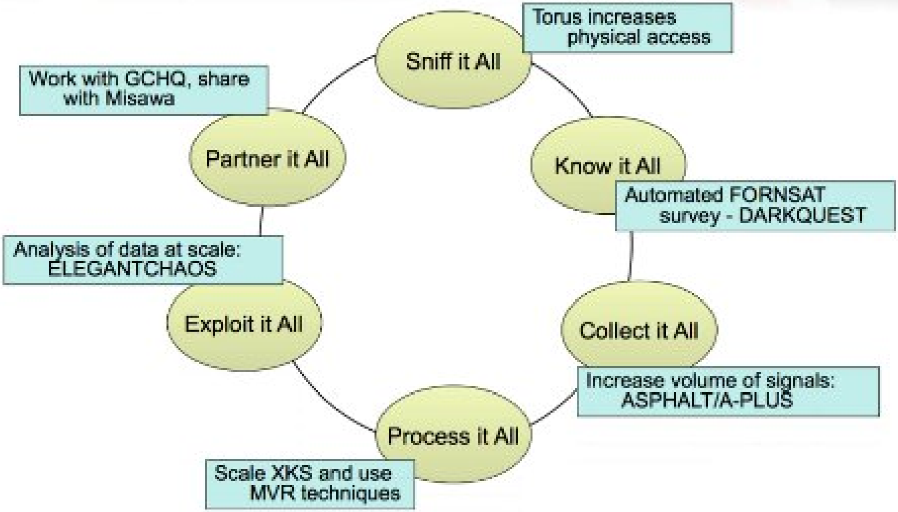
Algebraic topology of data sets

“What’s this?” I ask:



[extracted from article by Vin de Silva]

New Collection Posture



DNA knotting

From the website of Dorothy Buck (Imperial):

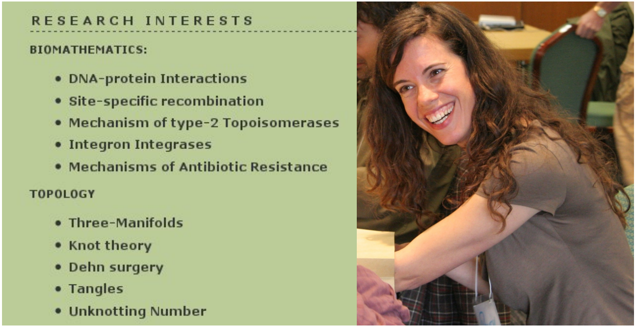


FIGURE 1. In these examples the recombinase complex B meets the substrate in the two crossover sites (highlighted in black).

[source: Buck and Flapan]

A few other applications of mathematics

- Algebraic topology and real algebraic geometry for robotics [link]
- Group-theoretic classification of insect gaits [link]
- Quantum field theory can predict patterns of biodiversity [link]

Applied maths
can help
pure maths

Random matrices and the Riemann zeta function

The year: 1972.

The scene: Afternoon tea at the Institute for Advanced Study, Princeton.

Freeman Dyson, dapper British physicist: 'So tell me, Montgomery, what have you been up to?'

Hugh Montgomery, boyish American mathematician: 'Well, lately I've been looking into the distribution of the zeros of the Riemann zeta function.'

Dyson: 'Yes? And?'

Montgomery: 'It seems the two-point correlations go as. . . ' *(turning to write on a nearby blackboard)*

Dyson: Extraordinary! Do you realize that's the pair-correlation function for the eigenvalues of a random Hermitian matrix? It's also a model of the energy levels in a heavy nucleus — say uranium 238.

[adapted from web page by Bob McLeod]

Random matrices and the Riemann zeta function

[source: Bob McLeod]

Two more examples of applied helping pure

- The *Gruppenpest* (plague of groups) [link]

- My collaborator Mark Meckes: [link]

We end this section by considering a quantity related to magnitude which is in some ways better behaved. For a compact (not necessarily positive definite) metric space A, the **maximum diversity** of A is

$$|A|_+ = \sup_{\mu \in P(A)} \left(\int \int e^{-d(a,b)} \, d\mu(a) \, d\mu(b) \right)^{-1}, \qquad (4.3)$$

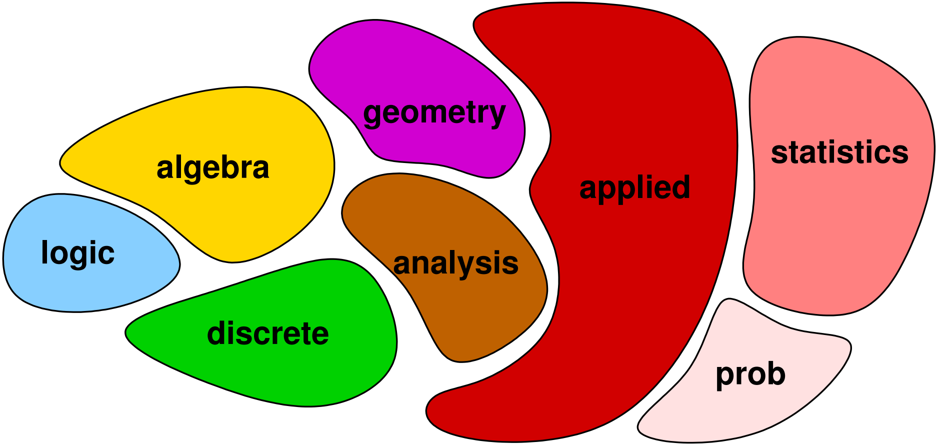
where $P(A)$ denotes the space of Borel probability measures on A. By renormalization, this is simply what one obtains by restricting the supremum in (3.5) to positive measures; thus we trivially have

$$|A|_+ \leq |A| \qquad (4.4)$$

for any compact PDMS A. The name stems from the following interpretation of the quantity

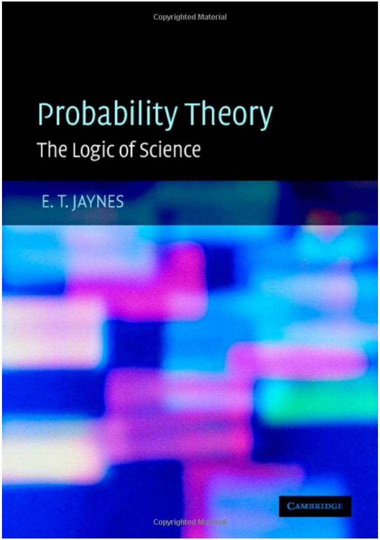
Prepare to rewire your brain

We all have a mental map of mathematics. . .



. . . but it can be misleading.

Statistical inference as a branch of logic



If A is true, then B becomes more plausible

B is true

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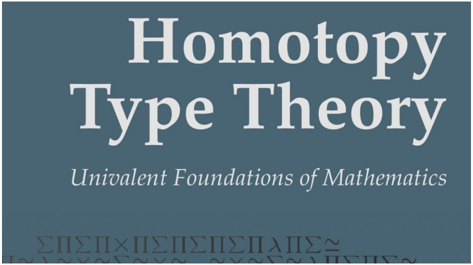
therefore, A becomes more plausible.

[Jaynes link]

Knowing unusual combinations of subjects gives you an advantage

Logic + topology + computer programming



Peter Aczel	Eric Finster	Alvaro Pelayo
Benedikt Ahrens	Daniel Grayson	Andrew Polonsky
Thorsten Altenkirch	Hugo Herbelin	Michael Shulman
Steve Awodey	André Joyal	Matthieu Sozeau
Bruno Barras	Dan Licata	Bas Spitters
Andrej Bauer	Peter Lumsdaine	Benno van den Berg
Yves Bertot	Assia Mahboubi	Vladimir Voevodsky
Marc Bezem	Per Martin-Löf	Michael Warren
Thierry Coquand	Sergey Melikhov	Noam Zeilberger

re were also the following students, whose participation was no less valuable.

Carlo Angiuli	Guillaume Brunerie	Egbert Rijke
Anthony Bordg	Chris Kapulkin	Kristina Sojakova

ddition, there were the following short- and long-term visitors, including student visit
se contributions to the Special Year were also essential.

Jeremy Avigad	Richard Garner	Nuo Li
Cyril Cohen	Georges Gonthier	Zhaohui Luo
Robert Constable	Thomas Hales	Michael Nahas
Pierre-Louis Curien	Robert Harper	Erik Palmgren
Peter Dybjer	Martin Hofmann	Emily Riehl
Martín Escardó	Pieter Hofstra	Dana Scott
Kuen-Bang Hou	Joachim Kock	Philip Scott
Nicola Gambino	Nicolai Kraus	Sergei Soloviev

Theorem	Status
$\pi_1(S^1)$	✔✔
$\pi_{k<n}(S^n)$	✔✔
long-exact-sequence of homotopy groups	✔✔
total space of Hopf fibration is S^3	✔
$\pi_2(S^2)$	✔✔
$\pi_3(S^2)$	✔
$\pi_n(S^n)$	✔✔
$\pi_4(S^3)$	✔
Freudenthal suspension theorem	✔✔
Blakers–Massey theorem	✔✔
Eilenberg–Mac Lane spaces $K(G,n)$	✔✔
van Kampen theorem	✔✔
covering spaces	✔✔
Whitehead's principle for n-types	✔✔

Table 8.2: Theorems from homotopy theory proved by hand (✔) and by computer (✔✔).

You can't learn everything yourself. . .

Books I've bought for one project

Elements of information theory
Geometric measure theory
Model theory
Matrix analysis
Measuring biological diversity
Fourier analysis in convex geometry
Lectures on functional equations and their applications
Convex geometry: the Brunn–Minkowski theory
Topics in matrix analysis
Advanced course in integral geometry and valuations
Introduction to geometric probability
Introduction to the theory of distributions
Metric structures for Riemannian and non-Riemannian spaces
A guide to distribution theory and Fourier transforms
Generalized functions (vols 1–3)
Functional analysis
On measures of information and their characterizations
Metric spaces, convexity and nonpositive curvature
Enumerative combinatorics vol 1
Mathematical foundations of information theory
Information and coding theory
Probability theory: the logic of science
Information theory, inference and learning algorithms
Inequalities
Geometry of sets and measures in Euclidean spaces
Mathematical theory of entropy
Probability with martingales
Real analysis
Fourier analysis
Fourier analysis
Maximum entropy and ecology
Real analysis and probability
Introducing genetics
A primer of ecology
Introduction to conservation ecology
Algebraic graph theory

You can't learn everything yourself...

...but you need to know enough to be able to communicate with your collaborators

Expect the unexpected!