# SMSTC Prospectus 2020-21 (draft)

Welcome to the Scottish Mathematical Sciences Training Centre. This document gives a brief overview for students and their supervisors of what SMSTC offers. We hope that it will be useful to you in planning the first few months of your PhD – and beyond!

SMSTC modules fall into two categories. **Core modules** are offered every year, and are formally assessed. **Supplementary modules** may change from year to year, and they may or may not be assessed. Supplementary modules are not necessarily more advanced than core modules, but often deal with a special topic of current research interest. All SMSTC students are welcome to attend both core and supplementary modules.

**What we offer is designed to be flexible.** SMSTC allows students to enrol in as many or as few modules as they like, in the first year or in later years of their PhDs; and to enrol in any module without taking the assessments. (However, many departments do have more specific requirements, so students are advised to discuss their choice of modules with their supervisors.) Students often gain most from taking a mixture of modules that relate directly to their research interests and modules that will **broaden their mathematical knowledge** in other areas – the latter may be more challenging, but also rewarding!

## How, what, and when

SMSTC lectures are delivered by **video conference using Zoom**; departments provide local tutorial support, which is not co-ordinated by SMSTC. Lecture notes and other material may be found on our **website**, www.smstc.ac.uk. To access this material, students will need to **register**; instructions can be found on the front page of the site.

The SMSTC year begins with the **online opening symposium**, which will be held on Wednesday 30 September and Thursday 1 October 2020. The symposium will include overviews of the modules, as well as sessions dealing with practical topics such as how to take tutorials and how to get a PhD.

**Semester 1** runs from Monday 5 October 2020 to Friday 11 December 2020, and **Semester 2** runs from Monday 11 January 2021 to Friday 19 March 2021.

Each core module will be assessed by one or more **assignments**. The deadline for the final assignment is typically two or three weeks after the end of the semester, and we aim to return grades a few weeks after that. Letters will be issued to students in late May confirming the modules that they have taken and the grades they have received.

More information about the structure of SMSTC and the **content of the modules** is available on the website. Please see especially the SMSTC Information for Students page,

https://www.smstc.ac.uk/information/students

# Modules available in 2020-21

## Core modules

The sixteen core modules are organised on four themes: Analysis [ANA]; Applications of Mathematics [AOM]; Probability and Statistics [PAS]; Structure and Symmetry [SAS]. The lectures for the core modules take place in the afternoons, Monday through Thursday, throughout each semester (see the detailed timetable on the final page). The titles of the core modules and their themes are as follows. *[The corresponding titles prior to 2017-18 are in italics.]*

**Semester 1**

| | |
|---|---|
| Groups, Rings and Modules [SAS] | *[Algebra 1]* |
| Dynamical Systems and Conservation Laws [ANA] | *[Applied Analysis and PDEs 1]* |
| Asymptotic and Analytical Methods [AOM] | *[Applied Mathematics Methods 1]* |
| Algebraic Topology [SAS] | *[Geometry and Topology 1]* |
| Continuum Mechanics [AOM] | *[Mathematical Models 1]* |
| Foundations of Probability [PAS] | *[Probability 1]* |
| Measure and Integration [ANA] | *[Pure Analysis 1]* |
| Regression and Simulation Methods [PAS] | *[Statistics 1]* |

**Semester 2**

| | |
|---|---|
| Algebras and Representation Theory [SAS] | *[Algebra 2]* |
| Elliptic and Parabolic PDEs [ANA] | *[Applied Analysis and PDEs 2]* |
| Numerical Methods [AOM] | *[Applied Mathematics Methods 2]* |
| Manifolds [SAS] | *[Geometry and Topology 2]* |
| Mathematical Biology and Physiology [AOM] | *[Mathematical Models 2]* |
| Stochastic Processes [PAS] | *[Probability 2]* |
| Functional Analysis [ANA] | *[Pure Analysis 2]* |
| Modern Regression and Bayesian Methods [PAS] | *[Statistics 2]* |

## Supplementary modules

In 2020-21 the following supplementary modules will be available. Lectures will take place in the mornings, Monday through Thursday, throughout each semester. However, some modules may not use the full ten weeks (see the detailed timetable on the final page, and the SMSTC website).

**Semester 1**

Combinatorics on Words
Finance, Risk, Asset-Pricing, Credit Scoring etc. (MAC-MIGS)
History of Mathematics
Geometry of Gauge Fields
Mathematical Foundations of Elasticity Theory (MAC-MIGS)

**Semester 2**

Classical and Quantum Integrable Systems
Incidence Geometry, Continuous and Discrete (MAC-MIGS; tbc)
Inverse Problems
Mathematics for the future energy industry (MAC-MIGS; tbc)
Variational methods for PDEs and nonlocal problems (MAC-MIGS)

# Which modules are suitable for me?

Every student taking SMSTC will have a different academic background and interests, so modules are not labelled as “introductory” or “advanced”. The following list of expected prior knowledge should give you an idea of where each module starts relative to what you've already studied. Before making any choices, though, you should also look at the syllabus (see the module page on the website) and if possible attend the Perth symposium where you will be able to speak to the module leader.

## Core modules: expected prior knowledge

**Algebraic Topology.** A working knowledge of metric and topological spaces; linear algebra (vector spaces, linear maps and quotient vector spaces); group theory (groups and group actions).

**Algebras and Representation Theory.** The notion of a module and related concepts; basics on Noetherian and Artinian modules; some commutative algebra, in particular the notion of a principal ideal domain.

**Asymptotic and Analytical Methods.** Basic ODEs (first-order separable and first- and second-order linear equations); single- and multivariable calculus; Taylor’s theorem; linear algebra; contour integration including Cauchy's theorem.

**Continuum Mechanics.** Introductory courses on ODEs, PDEs, vector calculus and basic linear algebra.

**Dynamical Systems and Conservation Laws.** Undergraduate-level ODEs, single- and multivariable real analysis, and linear algebra.

**Elliptic and Parabolic PDEs.** Undergraduate-level ODEs, single- and multivariable real analysis, and linear algebra.

**Foundations of Probability.** Elements of mathematical analysis, linear algebra and combinatorics at undergraduate level.

**Functional Analysis.** Undergraduate analysis: sequences, series, pointwise and uniform convergence. Metric space topology: at least in $R^d$, continuity of functions, open, closed and compact sets. Countable sets. Some of the examples draw upon the measure theory from the “Measure and Integration” module.

**Groups, Rings and Modules.** Basic linear algebra; definitions and examples of groups, rings, fields; basic algebra concepts such as homomorphisms; basic notions of group theory – permutations, symmetric groups, Lagrange's theorem, normal subgroups and factor groups.

**Manifolds.** A working knowledge of metric spaces; linear algebra (vector spaces, linear maps and quotient vector spaces); group theory (groups and group actions); vector calculus (differentiable map, Jacobian matrix and div/grad/curl); topological spaces and continuous functions; the fundamental group; covering spaces; the classification of surfaces; homology.

**Mathematical Biology and Physiology.** Undergraduate-level knowledge of ODEs, PDEs, vector calculus and basic linear algebra.

**Measure and Integration.** Undergraduate analysis: sequences, series, pointwise and uniform convergence. Metric space topology: at least in $R^d$, continuity of functions, open, closed and compact sets. Countable sets.

**Modern Regression and Bayesian Methods.** The “Regression and Simulation Methods” module or equivalent.

**Numerical Methods.** Basic ODEs (in particular first order separable and first- and second-order linear equations); single- and multivariable calculus; Taylor's theorem; and linear algebra. Prior knowledge of Matlab or Octave would be helpful.

**Regression and Simulation Methods.** Basic concepts in: probability (elementary probability distributions); statistics (ideas of estimation, confidence intervals, hypothesis tests); calculus. The level required in these areas would usually be provided in a first undergraduate course.

**Stochastic Processes.** Elements of mathematical analysis, linear algebra and combinatorics at undergraduate level. Probability theory, either at undergraduate level or from the "Foundations of Probability" module.

## Supplementary modules: expected prior knowledge

**Classical and Quantum Integrable Systems.** Some familiarity with manifolds and differential calculus on manifolds; groups and group actions. Lie groups and Lie algebras will also be needed, but the necessary material will be reviewed in the first lecture.

**Combinatorics on Words.** This module is intended for students with an interest in discrete mathematics. No prior knowledge is required.

**Finance, Risk, Asset-Pricing, Credit Scoring etc.** The course focuses on the finance for mathematics and topics in probability, statistics and stochastic processes are assumed. However, if a student has not taken courses in these subjects, they can contact the lecturer to discuss.

**Geometry of Gauge Fields.** Students taking this course should have some background in geometry and topology (definition of a manifold) and group theory, but no knowledge of Lie groups or gauge theory will be assumed.

**History of Mathematics.** Calculus, algebra, and analysis at the level of undergraduate understanding typically presumed of a postgraduate student in mathematical fields, but the discussion will be historical rather than mathematical so don't worry if you are not completely confident in all the details.

**Inverse Problems.** Solid linear algebra, probability and statistics and differential equations skills will be expected. No prior knowledge of any SMSTC level courses will be expected.

**Mathematical Foundations of Elasticity Theory.** No prior knowledge of elasticity theory is required.

# Timetables

## Semester 1

| | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| **Supp.** early morning | Mathematical Foundations of Elasticity Theory* (0930-1130) | | | Finance, Risk, etc.* (0930-1100) | |
| **Supp.** late morning | | | Geometry of Gauge Fields (1115-1245) | Combinatorics on Words (1115-1245) | |
| **Core** 1300-1500 | Groups, Rings & Modules | Regression & Simulation Methods | Asymptotic & Analytical Methods | Algebraic Topology | |
| **Core** 1530-1730 | Foundations of Probability | Dynamical Systems & Conservation Laws | Measure & Integration | Continuum Mechanics | History of Mathematics |

## Semester 2 (incomplete)

| | Monday | Tuesday | Wednesday | Thursday |
|---|---|---|---|---|
| **Supplementary** early morning | | Variational Methods for PDEs* (0900-1100) | | Inverse Problems (0930-1130) |
| **Supplementary** late morning | | | | |
| **Core** 1300-1500 | Algebras & Representation Theory | Modern Regression & Bayesian Methods | Numerical Methods | Manifolds |
| **Core** 1530-1730 | Stochastic Processes | Elliptic & Parabolic PDEs | Functional Analysis | Mathematical Biology & Physiology |

Modules marked * are provided by MAC-MIGS.