SMSTC Draft Prospectus 2019-20

Welcome to the Scottish Mathematical Sciences Training Centre. This document gives a brief overview for students and their supervisors of what SMSTC offers. We hope that it will be useful to you in planning the first few months of your PhD – and beyond!

SMSTC modules fall into two categories. **Core modules** are offered every year, and are formally assessed. **Supplementary modules** may change from year to year, and they may or may not be assessed. Supplementary modules are not necessarily more advanced than core modules, but often deal with a special topic of current research interest. All SMSTC students are welcome to attend both core and supplementary modules.

What we offer is designed to be flexible. SMSTC allows students to enrol in as many or as few modules as they like, in the first year or in later years of their PhDs; and to enrol in any module without taking the assessments. (However, many departments do have more specific requirements, so students are advised to discuss their choice of modules with their supervisors.) Students often gain most from taking a mixture of modules that relate directly to their research interests and modules that will broaden their mathematical knowledge in other areas – the latter may be more challenging, but also rewarding!

How, what, and when

SMSTC lectures are delivered by **video conference**; departments provide local tutorial support, which is not co-ordinated by SMSTC. Lecture notes and other material may be found on our **website**, www.smstc.ac.uk. To access this material, students will need to **register**; instructions can be found on the front page of the site.

The SMSTC year begins with the **opening symposium**, which will be held at the Dewars Centre in Perth, on Wednesday 2 and Thursday 3 October 2019. The symposium will include overviews of the modules, as well as sessions dealing with practical topics such as how to take tutorials and how to get a PhD. This is followed on 4 October by a **workshop for postgraduates who teach**; the workshop is run by the Institute for Mathematics and its Applications rather than SMSTC, but we endorse it and would encourage students to attend. Details can be found at

https://ima.org.uk/12239/supporting-postgraduates-who-teach-mathematics-statistics-edinburgh-2/

Semester 1 runs from Monday 7 October 2019 to Friday 13 December 2019, and **Semester 2** runs from Monday 6 January 2020 to Friday 13 March 2020.

Each core module will be assessed by one or more **assignments**. The deadline for the final assignment is typically two or three weeks after the end of the semester, and we aim to return grades a few weeks after that. Letters will be issued to students in late May confirming the modules that they have taken and the grades they have received.

More information about the structure of SMSTC and the **content of the modules** is available on the website. Please see especially the SMSTC Information for Students page,

https://www.smstc.ac.uk/information/students

Modules available in 2019-20

Core modules

The sixteen core modules are organised on four themes: Analysis [ANA]; Applications of Mathematics [AOM]; Probability and Statistics [PAS]; Structure and Symmetry [SAS]. The lectures for the core modules take place in the afternoons, Monday through Thursday, throughout each semester (see the detailed timetable on the final page). The titles of the core modules and their themes are as follows. *[The corresponding titles prior to 2017-18 are in italics.]*

Semester 1

Groups, Rings and Modules [SAS]	*[Algebra 1]*
Dynamical Systems and Conservation Laws [ANA]	*[Applied Analysis and PDEs 1]*
Asymptotic and Analytical Methods [AOM]	*[Applied Mathematics Methods 1]*
Algebraic Topology [SAS]	*[Geometry and Topology 1]*
Continuum Mechanics [AOM]	*[Mathematical Models 1]*
Foundations of Probability [PAS]	*[Probability 1]*
Measure and Integration [ANA]	*[Pure Analysis 1]*
Regression and Simulation Methods [PAS]	*[Statistics 1]*

Semester 2

Algebras and Representation Theory [SAS]	*[Algebra 2]*
Elliptic and Parabolic PDEs [ANA]	*[Applied Analysis and PDEs 2]*
Numerical Methods [AOM]	*[Applied Mathematics Methods 2]*
Manifolds [SAS]	*[Geometry and Topology 2]*
Mathematical Biology and Physiology [AOM]	*[Mathematical Models 2]*
Stochastic Processes [PAS]	*[Probability 2]*
Functional Analysis [ANA]	*[Pure Analysis 2]*
Modern Regression and Bayesian Methods [PAS]	*[Statistics 2]*

Supplementary modules

In 2019-20 the following supplementary modules will be available. Lectures will take place in the mornings, Monday through Thursday, throughout each semester. However, some modules may not use the full ten weeks (see the detailed timetable on the final page, and the SMSTC website).

Semester 1

Advanced PDE I
Convergence of Probability Measures and its Applications
Ethics in Mathematics
Galois Theory of Commutative Rings
Stochastic Networks and Heavy Tails
Stochastic PDEs: Applications and Numerics

Semester 2

Advanced PDE II
Algebraic Statistics
Classical and Quantum Integrable Systems
Solving Singular SPDEs with Regularity Structures

Which modules are suitable for me?

Every student taking SMSTC will have a different academic background and interests, so modules are not labelled as “introductory” or “advanced”. The following list of expected prior knowledge should give you an idea of where each module starts relative to what you've already studied. Before making any choices, though, you should also look at the syllabus (see the module page on the website) and if possible attend the Perth symposium where you will be able to speak to the module leader.

Core modules: expected prior knowledge

Algebraic Topology. A working knowledge of metric and topological spaces; linear algebra (vector spaces, linear maps and quotient vector spaces); group theory (groups and group actions).

Algebras and Representation Theory. The notion of a module and related concepts; basics on Noetherian and Artinian modules; some commutative algebra, in particular the notion of a principal ideal domain.

Asymptotic and Analytical Methods. Basic ODEs (first-order separable and first- and second-order linear equations); single- and multivariable calculus; Taylor’s theorem; linear algebra; contour integration including Cauchy's theorem.

Continuum Mechanics. Introductory courses on ODEs, PDEs, vector calculus and basic linear algebra.

Dynamical Systems and Conservation Laws. Undergraduate-level ODEs, single- and multivariable real analysis, and linear algebra.

Elliptic and Parabolic PDEs. Undergraduate-level ODEs, single- and multivariable real analysis, and linear algebra.

Foundations of Probability. Elements of mathematical analysis, linear algebra and combinatorics at undergraduate level.

Functional Analysis. Undergraduate analysis: sequences, series, pointwise and uniform convergence. Metric space topology: at least in R^d, continuity of functions, open, closed and compact sets. Countable sets. Some of the examples draw upon the measure theory from the “Measure and Integration” module.

Groups, Rings and Modules. Basic linear algebra; definitions and examples of groups, rings, fields; basic algebra concepts such as homomorphisms; basic notions of group theory – permutations, symmetric groups, Lagrange's theorem, normal subgroups and factor groups.

Manifolds. A working knowledge of metric spaces; linear algebra (vector spaces, linear maps and quotient vector spaces); group theory (groups and group actions); vector calculus (differentiable map, Jacobian matrix and div/grad/curl); topological spaces and continuous functions; the fundamental group; covering spaces; the classification of surfaces; homology.

Mathematical Biology and Physiology. Undergraduate-level knowledge of ODEs, PDEs, vector calculus and basic linear algebra.

Measure and Integration. Undergraduate analysis: sequences, series, pointwise and uniform convergence. Metric space topology: at least in R^d, continuity of functions, open, closed and compact sets. Countable sets.

Modern Regression and Bayesian Methods. The “Regression and Simulation Methods” module or equivalent.

Numerical Methods. Basic ODEs (in particular first order separable and first- and second-order linear equations); single- and multivariable calculus; Taylor's theorem; and linear algebra. Prior knowledge of Matlab or Octave would be helpful.

Regression and Simulation Methods. Basic concepts in: probability (elementary probability distributions); statistics (ideas of estimation, confidence intervals, hypothesis tests); calculus. The level required in these areas would usually be provided in a first undergraduate course.

Stochastic Processes. Elements of mathematical analysis, linear algebra and combinatorics at undergraduate level. Probability theory, either at undergraduate level or from the "Foundations of Probability" module.

Supplementary modules: expected prior knowledge

Advanced PDE I. Rigorous multivariable calculus (continuity, differentiability, chain rule, integration). Metric spaces, Banach spaces, Hilbert space, weak/strong convergence. Vector calculus, Green's formula, (normal, tangent/vectors, parametrisation of surfaces and curves).

Advanced PDE II. Rigorous multivariable calculus (continuity, differentiability, chain rule, integration). Metric spaces, Banach spaces, Hilbert space, weak/strong convergence. Vector calculus, Green's formula, (normal, tangent/vectors, parametrisation of surfaces and curves). Some knowledge of Sobolev spaces and elementary functional analysis an advantage.

Algebraic Statistics. Some familiarity with algebra, definitions and examples of groups, rings, fields, ideal (such as an introductory UG course in Algebra or Algebraic Geometry). Definitions and examples of probability distributions, likelihood function (such as an introductory UG course in Probability or Statistics).

Classical and Quantum Integrable Systems. Some familiarity with manifolds and differential calculus on manifolds; groups and group actions. Lie groups and Lie algebras will also be needed, but the necessary material will be reviewed in the first lecture.

Convergence of Probability Measures and its Applications. TBC.

Ethics in Mathematics. No prior knowledge is required.

Galois Theory of Commutative Rings. Standard undergraduate algebra including Galois Theory. Previous exposure to more advanced ring theory, algebraic number theory and homological algebra would be useful.

Solving Singular SPDEs with Regularity Structures. TBC.

Stochastic Networks and Heavy Tails. Previous SMSTC modules: Foundations of Probability; Stochastic Processes.

Stochastic PDEs: Applications and Numerics. TBC.

Full timetables (draft)

Semester 1

	Monday	Tuesday	Wednesday	Thursday
Supplementary early morning		* Stochastic Networks and Heavy Tails [09:00-10:45]	* Advanced PDE I [Wks 1-5: 09:30-11:30. Wks 6-10: 10:00-11:45.]	* Convergence of Probability Measures and its Applications [09:30-11:30]
Supplementary late morning	Galois Theory of Commutative Rings [11:15-12:45]	Ethics in Mathematics [Wks 1, 4, 7, 8, 9: 11:00-12:45]	* Stochastic PDEs: Applications and Numerics [11:50-12:45]	
Core 1300-1500	Groups, Rings & Modules	Regression & Simulation Methods	Asymptotic & Analytical Methods	Algebraic Topology
Core 1530-1730	Foundations of Probability	Dynamical Systems & Conservation Laws	Measure & Integration	Continuum Mechanics

Semester 2

	Monday	Tuesday	Wednesday	Thursday
Supplementary early morning			* Solving Singular SPDEs with Regularity Structures [09:00-11:00]	* Advanced PDE II [09:00-11:00]
Supplementary late morning			Classical and Quantum Integrable Systems [11:15-12:45]	Algebraic Statistics [11:15-12:45]
Core 1300-1500	Algebras & Representation Theory	Modern Regression & Bayesian Methods	Numerical Methods	Manifolds
Core 1530-1730	Stochastic Processes	Elliptic & Parabolic PDEs	Functional Analysis	Mathematical Biology & Physiology

Modules marked * are provided by MIGSAA.