SMSTC Prospectus 2017-18

Welcome to the Scottish Mathematical Sciences Training Centre. This document gives a brief overview for students and their supervisors of what SMSTC offers. We hope that it will be useful to you in planning the first few months of your PhD – and beyond!

SMSTC modules fall into two categories. **Core modules** are offered every year, and are formally assessed. **Supplementary modules** may change from year to year, and they may or may not be assessed. Supplementary modules are not necessarily more advanced than core modules, but often deal with a special topic of current research interest. All SMSTC students are welcome to attend both core and supplementary modules.

What we offer is designed to be flexible. SMSTC allows students to enrol in as many or as few modules as they like, in the first year or in later years of their PhDs; and to enrol in any module without taking the assessments. (However, many departments do have more specific requirements, so students are advised to discuss their choice of modules with their supervisors.) Students often gain most from taking a mixture of modules that relate directly to their research interests and modules that will broaden their mathematical knowledge in other areas – the latter may be more challenging, but also rewarding!

How, what, and when

SMSTC lectures are delivered by **video conference**; departments provide local tutorial support, which is not co-ordinated by SMSTC. Lecture notes and other material may be found on our **website**, www.smstc.ac.uk. To access this material, students will need to **register**; instructions can be found on the front page of the site.

The SMSTC year begins with the **opening symposium**, which will be held at the Dewars Centre in Perth, on Wednesday 4 and Thursday 5 October 2017. The symposium will include overviews of the modules, as well as sessions dealing with practical topics such as how to take tutorials and how to get a PhD.

Semester 1 runs from Monday 9 October 2017 to Friday 15 December 2017, and **Semester 2** runs from Monday 8 January 2018 to Friday 16 March 2018.

Each core module will be assessed by one or more **assignments**. The deadline for the final assignment is typically two or three weeks after the end of the semester, and we aim to return grades a few weeks after that. Letters will be issued to students in late May confirming the modules that they have taken and the grades they have received.

More information about the structure of SMSTC and the **content of the modules** is available on the website. Please see especially the Information for Students page,

https://www.smstc.ac.uk/about_us/information/student_information

Available modules in 2017-18

Core modules

The lectures for the sixteen core modules take place in the afternoons, Monday through Thursday, throughout each semester (see the detailed timetable on the final page). The titles of the core modules have changed this year, although the content generally remains the same. The new titles *[and old titles]* are as follows.

Semester 1

Groups, Rings and Modules	*[Algebra 1]*
Dynamical Systems and Conservation Laws	*[Applied Analysis and PDEs 1]*
Asymptotic and Analytical Methods	*[Applied Mathematics Methods 1]*
Algebraic Topology	*[Geometry and Topology 1]*
Continuum Mechanics	*[Mathematical Models 1]*
Foundations of Probability	*[Probability 1]*
Measure and Integration	*[Pure Analysis 1]*
Regression and Simulation Methods	*[Statistics 1]*

Semester 2

Algebras and Representation Theory	*[Algebra 2]*
Elliptic and Parabolic PDEs	*[Applied Analysis and PDEs 2]*
Numerical Methods	*[Applied Mathematics Methods 2]*
Manifolds	*[Geometry and Topology 2]*
Mathematical Biology and Physiology	*[Mathematical Models 2]*
Stochastic Processes	*[Probability 2]*
Functional Analysis	*[Pure Analysis 2]*
Modern Regression and Bayesian Methods	*[Statistics 2]*

Supplementary modules

In 2017-18 the following supplementary modules will be available. Lectures will take place in the mornings, Monday through Thursday, throughout each semester. However, some modules may not use the full ten weeks (see the detailed timetable on the final page, and the SMSTC website).

Semester 1

Advanced PDE 1
Analysis and Numerics of Stochastic PDEs
Stochastic Integration
The mod 2 Steenrod Algebra in Theory and in Practice

Semester 2

Advanced PDE 2 (to be confirmed)
Diffusion Processes
Homogenisation I: Multiscale Modelling and Analysis of Physical and Biological Processes
Homogenisation II: Stochastic Problems
What is Numerical Analysis?

Which modules are suitable for me?

Every student taking SMSTC will have a different academic background and interests, so modules are not labelled as “introductory” or “advanced”. The following list of expected prior knowledge should give you an idea of where each module starts relative to what you've already studied. Before making any choices, though, you should also look at the syllabus (see the module page on the website) and if possible attend the Perth symposium where you will be able to speak to the module leader.

Core modules: expected prior knowledge

Algebraic Topology. A working knowledge of metric and topological spaces; linear algebra (vector spaces, linear maps and quotient vector spaces); group theory (groups and group actions).

Algebras and Representation Theory. The notion of a module and related concepts; basics on Noetherian and Artinian modules; some commutative algebra, in particular the notion of a principal ideal domain.

Asymptotic and Analytical Methods. Basic ODEs (first-order separable and first- and second-order linear equations); single- and multivariable calculus; Taylor’s theorem; linear algebra; contour integration including Cauchy's theorem.

Continuum Mechanics. Introductory courses on ODEs, PDEs, vector calculus and basic linear algebra.

Dynamical Systems and Conservation Laws. Undergraduate-level ODEs, single- and multivariable real analysis, and linear algebra.

Elliptic and Parabolic PDEs. Undergraduate-level ODEs, single- and multivariable real analysis, and linear algebra.

Foundations of Probability. Elements of mathematical analysis, linear algebra and combinatorics at undergraduate level.

Functional Analysis. Undergraduate analysis: sequences, series, pointwise and uniform convergence. Metric space topology: at least in R^d, continuity of functions, open, closed and compact sets. Countable sets. Some of the examples draw upon the measure theory from the “Measure and Integration” module.

Groups, Rings and Modules. Basic linear algebra; definitions and examples of groups, rings, fields; basic algebra concepts such as homomorphisms; basic notions of group theory – permutations, symmetric groups, Lagrange's theorem, normal subgroups and factor groups.

Manifolds. A working knowledge of metric spaces; linear algebra (vector spaces, linear maps and quotient vector spaces); group theory (groups and group actions); vector calculus (differentiable map, Jacobian matrix and div/grad/curl); topological spaces and continuous functions; the fundamental group; covering spaces; the classification of surfaces; homology.

Mathematical Biology and Physiology. Introductory courses on ODEs, PDEs, vector calculus and basic linear algebra.

Measure and Integration. Undergraduate analysis: sequences, series, pointwise and uniform convergence. Metric space topology: at least in R^d, continuity of functions, open, closed and compact sets. Countable sets.

Modern Regression and Bayesian Methods. The “Regression and Simulation Methods” module or equivalent.

Numerical Methods. Basic ODEs (in particular first order separable and first- and second-order linear equations); single- and multivariable calculus; Taylor’s theorem; and linear algebra. Prior knowledge of Matlab or Octave would be helpful.

Regression and Simulation Methods. Basic concepts in: probability (elementary probability distributions); statistics (ideas of estimation, confidence intervals, hypothesis tests); calculus. The level required in these areas would usually be provided in a first undergraduate course.

Stochastic Processes. Elements of mathematical analysis, linear algebra and combinatorics at undergraduate level. Probability theory, either at undergraduate level or from the “Foundations of Probability” module.

Supplementary modules: expected prior knowledge

Advanced PDE I. Rigorous multivariable calculus (continuity, differentiability, chain rule, integration). Metric spaces, Banach spaces, Hilbert space, weak/strong convergence. Vector calculus, Green's formula, (normal, tangent/vectors, parametrisation of surfaces and curves).

Advanced PDE II. Familiarity with Banach and Hilbert spaces, dual spaces, and weak and strong convergence. The module “Advanced PDE I” is suggested but not required.

Analysis and Numerics of Stochastic PDEs. Basic notions and results of probability theory and stochastic analysis. Elements of functional analysis concerning linear operators on Hilbert and Banach spaces.

Diffusion Processes. Basic functional analysis and PDE theory, basic stochastic calculus (at least Brownian Motion, elements of SDE theory and Ito formula), some familiarity with the notion of Markov Process.

Homogenisation I: Multiscale Modelling and Analysis of Physical and Biological Processes. Some knowledge of Sobolev spaces and PDEs.

Homogenisation II: Stochastic Problems. This course is a follow up to Homogenisation I, but those interested in stochastic averaging techniques will be able to follow it without having attended the first course. Useful but not required: “Advanced PDE I” and basic knowledge in probability theory and SDEs and associated Kolmogorov equations. Also useful: basic knowledge of measure and integration theory, and functional analysis; basic knowledge of Galerkin/finite element approximations and finite difference methods.

Steenrod Algebra. A basic knowledge of cohomology with coefficients for spaces.

Stochastic Integration. SMSTC “Stochastic Processes” or equivalent.

What is Numerical Analysis? A previous course in either PDEs or their numerical analysis.

Full timetables

Semester 1

	Monday	Tuesday	Wednesday	Thursday	Friday
Supplementary early morning	09:00-10:30 Advanced PDE 1 (TBC)			09:00-10:15 Stochastic Integration	
Supplementary late morning		11:00-12:50 Stochastic PDEs	11:00-12:30 Steenrod Algebra (weeks 3-11)		
Core 13:00-15:00	Groups, Rings & Modules	Regression & Simulation Methods	Asymptotic & Analytical Methods	Algebraic Topology	
Core 15:30-17:30	Foundations of Probability	Dynamical Systems & Conservation Laws	Measure & Integration	Continuum Mechanics	

Semester 2

	Monday	Tuesday	Wednesday	Thursday	Friday
Supplementary early morning		09:00-10:30 Diffusion Processes (TBC)	*09:00-10:50 Advanced PDE 2 (TBC)*	09:00-11:00 Homogenisation II	
Supplementary late morning		10:45-12:45 What is Numerical Analysis?		11:15-12:45 Homogenisation I (TBC)	
Core 13:00-15:00	Algebras & Representation Theory	Modern Regression & Bayesian Methods	Numerical Methods	Manifolds	
Core 15:30-17:30	Stochastic Processes	Elliptic & Parabolic PDEs	Functional Analysis	Mathematical Biology & Physiology	

Some changes to this schedule may still be made, especially to the supplementary modules.

The module in *green italics* (Advanced PDE 2) may or may not be made available on the SMSTC network. Like most of the supplementary modules, it is provided by the Maxwell Institute Graduate School in Analysis and its Applications (MIGSAA).