Homological algebra

Pavel Safronov

September 2022

Why homological algebra?

- **Topology**. Universal coefficient theorem: if X is a topological space and A an abelian group, one can express $H_i(X; A)$ in terms of $H_i(X; \mathbf{Z})$ using the *Tor* functor.
- **Algebraic geometry**. Classification of vector bundles/coherent sheaves on $\mathbf{CP}^n$ is difficult. The description of the *derived category* of coherent sheaves on $\mathbf{CP}^n$ can be reduced to linear algebra.
- **Mathematical physics**. BRST and BV formalisms for quantizations of gauge theories are homological in nature.
- **Topological data analysis**. Persistent homology groups are closely related to the spectral sequence associated to a filtered complex.

Content of the course

- Abelian categories.
- Derived functors.
- Examples: group cohomology, Lie algebra cohomology, Hochschild cohomology.
- Spectral sequences.
- Derived categories.
- Simplicial objects and the Dold–Kan correspondence.
- DG categories.

We follow the textbooks by Weibel and Gelfand–Manin.

Structure

- Runs in Semester 2.
- 10 weekly 1.5-hour-long lectures.
- 2 homeworks.