

# SMSTC, Structure and Symmetry

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- ★ **Topology** (from the Greek topos, place, and logos, study) is concerned with the properties of a geometric object that are preserved under continuous deformations, such as stretching, twisting, crumpling and bending, but not tearing or gluing.

# Theme overview

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## *Semester 1*

### ★ **Groups, Rings and Modules**

- Louise Theran, **University of St. Andrews**  
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- Qingyuan Jiang, **University of Edinburgh**  
[qingyuan.jiang@ed.ac.uk](mailto:qingyuan.jiang@ed.ac.uk)

### ★ **Algebraic Topology**

- Alessandro Sisto, **Heriot-Watt University**  
[A.Sisto@hw.ac.uk](mailto:A.Sisto@hw.ac.uk)
- Dinakar Muthiah, **University of Glasgow**  
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# Theme overview

*Semester 2*

★ **Algebras and Representation Theory**

★ **Manifolds**

# Prerequisites

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## ★ **Groups, Rings and Modules**

- ▶ Basic linear algebra and basic algebra concepts.
  - Definitions and examples of groups, rings and fields.
- ▶ Basic notions of group theory.
  - Lagrange's theorem, normal subgroups and factor groups.

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## ★ **Algebraic Topology**

- ▶ A course in metric spaces or topological spaces.
- ▶ A course in group theory.
  - Group actions.
  - Finitely generated abelian groups.

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## ★ **Algebras and Representation Theory**

- ▶ The notion of a module and related concepts.
- ▶ Basics on noetherian and artinian modules.
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## ★ Manifolds

- ▶ Standard calculus courses.
  - Green's theorem.
- ▶ Basic courses in linear algebra.
  - Abstract vector space.

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- Representation Theory.
  - ▶ Representations and characters.
  - ▶ Orthogonality relations.
  - ▶ Induced representations.
  - ▶ Computing character tables.



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- (9) Euler characteristic, the Gauss-Bonnet Theorem for surfaces.

Enjoy the Theme!