Applications of Mathematics

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Overview of the theme

The theme can be divided into two broad categories:

- the formulation and analysis of *mathematical models*
- *methods or tools* needed in the process

Two modules per category; self-contained and independent.

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- **mathematical biology**

Not comprehensive, but illustrative of a range of approaches.

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Two complementory sets of methods:

- **asymptotic and analytic**
- **numerical**

Again not comprehensive, but illustrative. In both cases the aim is to develop *approximate* methods in a systematic, quantitative way.

Continuum Mechanics (semester 1)

Continuum mechanics describes how deformable media behave.

Huge range of applications (nano-technology to astrophysics).

Topics:

- **rational continuum mechanics:** construction of dynamical models of deformable media
- **fluid dynamics:** lubrication theory, aerofoils, hydrodynamic stability
- **non-Newtonian fluids:** fluid viscosity depends on internal stresses

Mathematical Biology and Physiology (semester 2)

Mathematical modelling in the life sciences: exciting and rapidly evolving!

Topics:

- **mathematical physiology:** cell-scale to macro-scale via homogenisation
- **population modelling:** epidemiology, evolution, pathogen-host interactions, age-structured models
- **mathematical oncology:** cancel modelling
- **bacterial resistance:** antibiotics

Asymptotic and Analytical Methods (semester 1)

Methods for problems typically involving a *small parameter*, $\varepsilon \ll 1$

Difference between $\varepsilon = 0$ and $\varepsilon \ll 1$ is often profound.

Often remarkably successful, even when they shouldn't be!

Topics:

- **multiple scales:** modulation and resonance
- **matched asymptotics:** boundary layers
- **WKBJLG theory:** rapidly oscillating solutions, ray theory
- **approximation of integrals:** Watson's lemma, steepest descents
- **intermediate asymptotics:** self-similarity of solutions
- **resummation:** techniques for series

Numerical Methods (semester 2)

Ways to solve ODEs and PDEs numerically.

Often there is a compromise between ease of implementation and efficiency.

Or between speed and accuracy.

Topics:

- **ODEs:** explicit and implicit methods (stability)
- **stochastic DEs:** an introduction
- **PDEs:** finite-difference and finite-element methods
- **linear algebra:** linear systems, eigenvalues etc.

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- complex variables

More details on the module pages.

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Assessment: two assignments per module

- mix of analytic and computational work
- normally at least two weeks to complete each