SMSTC: Supplementary (Advanced) courses

MIGSAA, Maxwell Institute

Perth 2 October 2019

MAXWELL INSTITUTE FOR MATHEMATICAL SCIENCES

SMSTC Core courses

Semester 1

- Algebraic Topology
- Asymptotic and Analytical Methods
- Continuum Mechanics
- Dynamical Systems and Conservation Laws
- Foundations of Probability
- Groups, Rings and Modules
- Measure and Integration
- Regression and Simulation Methods

Semester 2

- Algebras and Representation Theory
- Elliptic and Parabolic PDEs
- Functional Analysis
- Manifolds
- Mathematical Biology and Physiology
- Modern Regression and Bayesian Methods
- Numerical Methods
- Stochastic Processes

Supplementary (Advanced) courses: Semester 1

Advanced PDE I: Elliptic & parabolic PDE *Daniel Coutand & Istvan Gyongy,* *(20 hours, 15 credits)*

Wednesdays, 10:00am –11.45 am & 9.30 am – 11:30 am

Syllabus:

- *The model equation* $\Delta u - \lambda u = f$ *for constants* $\lambda > 0$
- *Sobolev spaces, generalised and Sobolev derivatives. Motivation and basic properties.*
- *Solvability of the model equation when* $f \in L_2$.
- *Solvability of elliptic PDEs on the whole space in Sobolev spaces.*
- *Equations in divergence form*
- *Elliptic Equations on domains. Interior regularity, Boundary regularity*
- *Parabolic equations, main examples, maximum principle*
- *Parabolic setting and Sobolev spaces*
- *Global in time solutions for nonlinear parabolic problems with small initial data*
- *Energy estimates*

Prerequisites: *rigorous multivariable calculus (continuity, differentiability, chain rule, integration); Metric spaces, Banach spaces, Hilbert space, weak/strong convergence; vector calculus, Green's formula*

Assessment: *4 (2+2) sets of homework.*

Text book(s):

1. *L.C. Evans, Partial Differential Equations, AMS*
2. *N.V. Krylov, Lectures on Elliptic and Parabolic Equations in Sobolev spaces, AMS Graduate Studies in Mathematics.*

Supplementary (Advanced) courses: Semester 1

<u>**Convergence of Probability Measures and Its Applications**</u> *Burak Buke,* *(20 hours, 15 credits)*

Thursday, 9:30am –11.30 am

Syllabus:

- *Basics of Weak Convergence (Measures on Metric Spaces Properties of Weak Convergence, Prokhorov?s Theorem*
- *The Space of Continuous Functions C (Uniform topology on C, Criteria for Compactness and Tightness, Donsker's Theorem and Brownian Motion*
- *The Space of Right-Continuous Functions (Space D) (Four Skorokhod Topologies, Tightness in D, Sequences with Continuous Limits, C-Tightness).*
- *Convergence of Markov Processes (Characterization of Markov Processes, Stone?s Lemma for Birth-Death processes, Convergence of Generators)*
- *Martingale Methods for Weak Convergence (Basic properties of Martingales, Quadratic Variations and Basic Inequalities, Martingale Central Limit Theorem, Martingale Problems for Markov Processes)*
- *Applications (Manufacturing and Service Industry, Telecommunication Networks, Mathematical Finance, Chemical Reactions, Mathematical Biology, Statistical Physics and Mean Field Limits)*

Assessment: *individual or group projects that focus on how weak convergence techniques are used for different applications*

Supplementary (Advanced) courses: Semester 1

Stochastic PDEs, Applications and Numerics *Istvan Gyongy,* *(10 hours, 15 credits)*

Wednesdays, 11:50am –12.45 am

Syllabus:

1. *Main results on solvability of linear SPDEs*
   - *Introduction: Examples of stochastic PDEs (SPDEs) arising in applications, SPDEs in nonlinear filtering and in population genetics*
   - *Stochastic processes with values in Sobolev spaces, and Ito formulas for their functions*
   - *Existence and uniqueness theorems in Sobolev spaces for SPDEs in the whole Euclidean space.*
   - *Stochastic Fubini theorem and Ito-Wentzell formula. Feynman-Kac formulas for PDEs and SPDEs*

2. *Numerical schemes for PDEs and SPDEs of parabolic type*
   - *Spatial discretisation, rate of convergence, accelerated schemes*
   - *Time discretisation, accuracy of implicit and explicit methods*
   - *Fully discretised schemes*
   - *Localisation error*
   - *Splitting up approximations, accelerated splitting up methods*
   - *Wong-Zakai approximations for SPDEs*

Prerequisites: *Basics of probability theory and stochastic analysis, elements of functional analysis*

Supplementary (Advanced) courses: Semester 2

Advanced PDE II: Hyperbolic Equations *John Ball & Pieter Blue,* *(20 hours, 15 credits)*

Thursdays, 9:00am –11.00 am

Syllabus:

- *The linear wave equation in bounded and unbounded domains, Huygen's principle, weak solutions and associated semigroup.*
- *Semilinear wave equations: local and global existence via variation of constants formula, damped equations and approach to equilibrium, finite-time blow-up.*
- *Introduction to quasilinear systems and more detailed treatment of scalar case.*
- *Energy methods for hyperbolic PDE. Uniqueness of C^2 solutions. Decay estimates and global existence for solutions above the energy norm.*
- *Strichartz estimates. Existence of solutions to quasilinear equations.*

Prerequisites: *Rigorous multivariable calculus (continuity, differentiability, chain rule, integration). Metric spaces, Banach spaces, Hilbert spaces, weak/strong convergence. Vector calculus, Green's formula, (normal/tangent vectors, parameterisation of surfaces and curves). Some knowledge of Sobolev spaces and elementary functional analysis an advantage.*

Assessment: *2 assignments*

Text book(s):

1. *F. John, Partial differential equations, 4th edition, Springer-Verlag, 1982.*
2. *T. Cazenave and A. Haraux,An introduction to semilinear evolution equations, Oxford University Press, 1998.*
3. *C. M. Dafermos, Hyperbolic conservation laws in continuum physics, Third edition, Grundlehren der mathematischen Wissenschaften Vol. 325, Springer-Verlag, 2010.*

Supplementary (Advanced) courses: Semester 2

Solving singular SPDEs with Regularity Structures *Yvain Bruned,* *(20 hours, 15 credits)*

Wednesdays, 9:00am –11.00 am

Syllabus: *The Regularity Structures introduced in [Hai14], inspired by Rough Paths have been able to solve several singular stochastic partial differential equations (SPDEs). These parabolic equations are formed of singular space-time noises and non-linearities depending on the solution and its derivatives. In this course, we will see how to solve these singular equations using this theory. We will first recall the basic definitions of a regularity structure and see different examples and applications. The main part of the course will be dedicated to the study of singular SPDEs but the main idea is to view regularity structures as a computational tool. We will also give a summary of some recent advances in this field explaining how these equations can be solved in a modern way using a black box introduced in [BHZ19, CH16, BCCH17]. A good reference is [FH14] and the aim of the course is to go beyond that reference by looking at the latest techniques available in the literature.*

Prerequisites: *No particular prerequisites are required. The course will start by introducing basic definitions. A previous course on stochastic analysis and a background in functional analysis can help.*

Assessment: *It can take various forms:*

- *Students can write essays to some related topics around this course.*
- *Typing lecture notes in latex for this course.*
- *Give one hour presentation on related topics.*

References:

BCCH17 *Y. Bruned, A. Chandra, I. Chevyrev, and M. Hairer. "Renormalising SPDEs in regularity structures", to appear in J. Eur. Math. Soc. arXiv:1711.10239.*

BHZ19 *Y. Bruned, M. Hairer, and L. Zambotti. "Algebraic renormalisation of regularity structures". Invent. Math. 215, no. 3, (2019), 1039?1156.*

CH16 *A. Chandra and M. Hairer. "An analytic BPHZ theorem for regularity structures". arXiv:1612.08138.*

FH14 *P. K. Friz and M. Hairer. "A Course on Rough Paths: With an Introduction to Regularity Structures". Springer Universitext. Springer, 2014, p. 252.*

Hai14 *M. Hairer. "A theory of regularity structures". Invent. Math. 198, no. 2, (2014), 269?504.*

Further non SMSTC advanced MIGSAA courses

1. Semester 1:
   - Additive Combinartorics, *Rob Fraser*. (Mondays 9:00–11:00).
   - Stochastic Networks and Heavy Tails, *Sergey Foss* (Tuesdays 9:00–10:45).
2. Semester 2:
   - Modern developments in Fourier analysis, *Jonathan Hickman*. (Tuesdays and Fridays 9:15–10:15).
   - Numerical Analysis of Partial Differential Equations, *Heiko Gimperlein*. (Tuesdays 10:30–12:30).
   - Rough Path Theory and Pathwise Well-posedness of Stochastic PDEs, *Hiro Oh & Oana Pocovnicu* (Wednesdays 11:15–12:45).