Supplementary modules

What's on offer

Almost all of this year's supplementary modules are in analysis...

...all except one!

That's

Central Simple Algebras and Galois Cohomology

taught by Adam Morgan (Glasgow) in Semester 2.

Essential prerequisites:
Commutative algebra (groups, rings, modules) and a first course in Galois theory.

Desirable prerequisites:
First course in representation theory. Prior familiarity with a small amount of algebraic geometry will be helpful but by no means necessary.

SMSTC: Supplementary (Advance) courses

MIGSAA, Maxwell Institute

Edinburgh 3 October 2018

HERIOT WATT UNIVERSITY

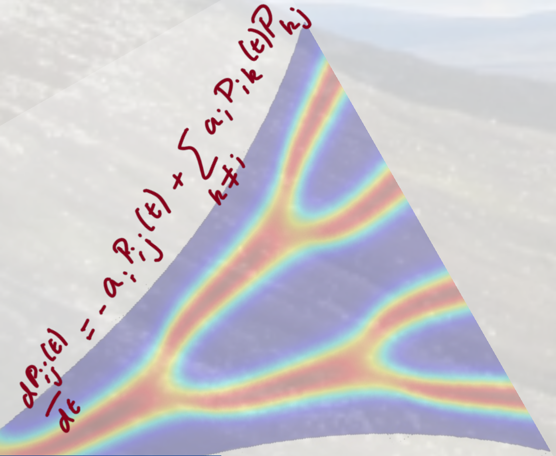
LMS-CMI Research School

29 April – 3 May 2019
ICMS, Edinburgh

“PDEs in Mathematical Biology: Modelling and Analysis”

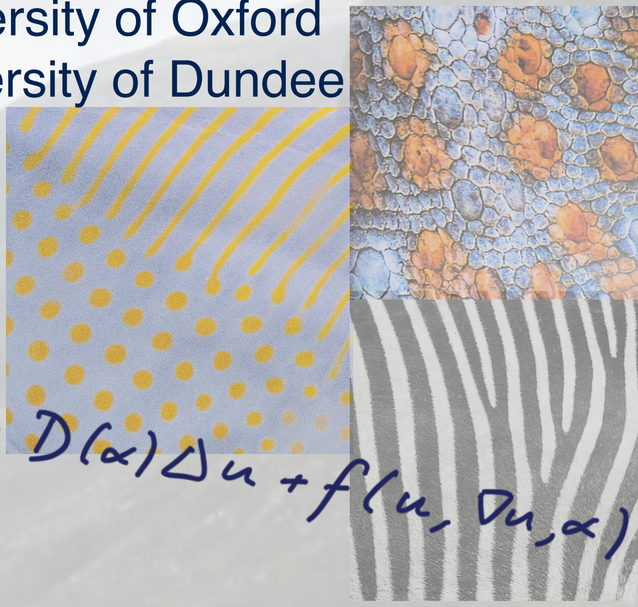
Lecturers
Dagmar Iber, ETH Zurich
Jonathan Potts, Sheffield University
Elaine Crooks, Swansea University
Benoit Perthame, Pierre et Marie Curie Univ.
Luigi Preziosi, Politecnico di Torino

Guest speakers
Angela Stevens, Muenster University
Alain Goriely, University of Oxford
Kees Weijier, University of Dundee



$$\frac{dP_{ij}(t)}{dt} = -a_i P_{ij}(t) + \sum_{k \neq i} a_i P_{ik}(t) P_{kj}(t)$$

$$dv = F(D^2u, Du)\,dt + H(Du) \circ dW_t$$



$$\frac{\partial u}{\partial t} = D(\alpha)\Delta u + f(u, \nabla u, \alpha)$$

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SMSTC Core courses

Semester 1

- Algebraic Topology
- Asymptotic and Analytical Methods
- Continuum Mechanics
- Dynamical Systems and Conservation Laws
- Foundations of Probability
- Groups, Rings and Modules
- Measure and Integration
- Regression and Simulation Methods

Semester 2

- Algebras and Representation Theory
- Elliptic and Parabolic PDEs
- Functional Analysis
- Manifolds
- Mathematical Biology and Physiology
- Modern Regression and Bayesian Methods
- Numerical Methods
- Stochastic Processes

MIGSAA Advanced Courses

Timetable: Semester One

	Monday	Tuesday	Wednesday	Thursday
Supplementary early morning	09:00 -10:30 Geometric MeasureTheory Jonas Azzam **NOT SMSTC** **5.46 Bayes Centre ***	09.00 – 10.30	09.00 – 10.30 Geometric Measure Theory Jonas Azzam **NOT SMSTC** **5.46 Bayes Centre ***	09:30-11.30 Brascamp-Lieb Tony Carbery **NOT SMSTC** **Various Venues ***
Supplementary late morning	11.15 – 12.45 Diffusion Processes Michela Ottobre **5.46 Bayes Centre**	11:00-12:50 Advanced PDE 1 Istvan Gyongy 5 weeks & Daniel Coutand 5 weeks) **5.46 Bayes Centre**	11:00-12:30 Homogenisation I Mariya Ptashnyk **5.46 Bayes Centre**	11.45-12.30

Timetable: Semester

	Monday	Tuesday	Wednesday	Thursday
Supplementary early morning	09:00-10:30 Spectral Theory and Differential Operators Lyonell Boutlon **to be confirmed** **5.46 Bayes Centre**	09:00-10:30 Probabilistic Representation for Linear and Nonlinear PDE David Siska & Lukasz Szpruch **NOT SMSTC** **5.46 Bayes Centre ***	09:00-10:50	09:00-10:45
Supplementary late morning		10:45-12:45 Variational methods for PDEs Heiko Gimperlein **5.46 Bayes Centre**	11:00-12:30	11:00-12:50 Two-dimensional statistical hydrodynamics Hiro Oh **NOT SMSTC** **5.46 Bayes Centre ***

*** Register via the course lecturer copying I.Hanlon@ed.ac.uk. Where there are various venues, information will be emailed out upon registration.**

All other courses are SMSTC, registration is via the SMSTC website https://smstc.ac.uk/supplementary_modules .

Supplementary (Advance) courses: Semester 1

<u>**Diffusion Processes**</u> *Michela Ottobre* (20 hours, 15 credits)

Mondays 11.15am – 12.45pm

Format: 1/2 of the course - taught lectures, the second part will be organised as reading-group style

Syllabus: The theory of time-homogeneous diffusion processes from the analysis standpoint.

- Markov Semigroups and their generators
- Ergodic Theory for Markov Semigroups
- Backward Kolmogorov and Fokker-Planck equation
- Reversible diffusions: spectral gap inequality, exponentially fast convergence to equilibrium
- Over and under-dumped Langevin equation

Flexible in topics for second half

Prerequisites: Basic probability theory, basic stochastic calculus (e.g. Ito formula), very basic SDE and PDE theory

Useful: Probability 1 SMSTC stream (and some of Probability 2)

Assessment: First part: question sheets

Second part: presentations and active participation

Supplementary (Advance) courses: Semester 1

Geometric Measure Theory *Jonas Azzam* (20 hours, 15 credits)

Monday & Wednesdays. 9:00am – 10.30am **(not available to SMSTC)**

Syllabus: Essential techniques from geometric measure theory. This field studies the geometry of sets and measures in Euclidean and metric spaces, motivated by problems from complex analysis, harmonic analysis in non-Euclidean spaces, PDEs.

- There is a rich family of sets of Lebesgue measure zero, but how can we compare their sizes in a meaningful way?
- Hausdorff measures
- What happens to the dimension of a set under transformations?
- What does the geometry of a measure behave like as I zoom in?
- Rectifiable sets (n-dimensional manifold with a weak differentiable structure (approximated by tangent planes almost everywhere))

Prerequisites: Only real analysis is required, although complex analysis or harmonic analysis would improve the experience.
Assessment: There will be weekly exercises on the material
P. Mattila, *Geometry of Sets and Measures in Euclidean Space*, 1995

Supplementary (Advance) courses: Semester 1

Advanced PDE I: Elliptic & parabolic PDE (20 hours, 15 cred.)

Daniel Coutand & Istvan Gyongy, *Tuesdays, 11:00 am – 12:30 pm*

Syllabus:

- Examples of elliptic equations, maximum and comparison principles
- Classical solutions, Bernstein estimate, applications.
- Schauder estimates
- Sobolev spaces, Weak solutions, Lax-Milgram
- Interior regularity, Boundary regularity
- Parabolic equations
- Global in time solutions for nonlinear parabolic problems
- Energy estimates

Prerequisites: rigorous multivariable calculus (continuity, differentiability, chain rule, integration); Metric spaces, Banach spaces, Hilbert space, weak/strong convergence; vector calculus, Green's formula

Assessment: 4 (2+2) sets of homework.

Text book: L.C. Evans, *Partial Differential Equations*, AMS

Supplementary (Advance) courses: Semester 2

Homogenization theory *Mariya Ptashnyk* (20 hours, 15 credits)

Wednesdays 11.00am – 12.30pm

Syllabus:

- Main methods of periodic homogenization
- Multiscale modelling and analysis of
  - fluid flow in porous media
  - transport and reaction processes in perforated domains
  - equations of linear elasticity and viscoelasticity
- Dual-porosity problem: modelling and multiscale analysis (transport and reaction processes in fractured media, in soil with porous particles, in cell tissues)
- Main ideas of Γ-, G- and H- convergences: multiscale analysis of nonlinear problems

Prerequisites: some knowledge of Sobolev spaces and PDEs

Assessment: 4 sets of homeworks/tutorials

D. Cioranescu, P. Donato, *An introduction to homogenization*, 1999

A. Bensoussan, J.L. Lions, G. Papanicolaou, *Asymptotic analysis for periodic structures*, 1978

Supplementary (Advance) courses: Semester 2

Brascamp-Lieb and Related Inequalities *Anthony Carbery*

Thursdays 09.30am –11.30pm **(not available to SMSTC)**

Syllabus:

- Young's convolution inequality, Brascamp-Lieb inequalities, Optimal mass transport and Barthe's proof of Lieb's theorem, Bennett-Carbery-Christ-Tao finiteness theorem, heat-flow method
- The geometric Brascamp-Lieb inequality of Keith Ball, Applications to convex geometry, Gaussian Brascamp-Lieb inequalities
- The Ornstein-Uhlenbeck semigroup, Nelson's inequality, hypercontractivity, log-Sobolev inequalities, Gaussian isoperimetry
- Regularity of the Brascamp-Lieb bound, Nonlinear Brascamp-Lieb inequalities, Applications in PDE, Connections to complexity theory and complete positivity in operator algebras

Format: Lectures + Students' Presentations
Prerequisites: Basic real variables and functional analysis, basic harmonic analysis, L^p-spaces and basic inequalities
Remark: Not designed for Year 1 students, however, a suitably qualified student in Year 1 could take it subject to the instructor's agreement.

Supplementary (Advance) courses: Semester 2

Spectral Theory *Lyonell Boulton* (20 hours, 15 credits)

Mondays 9.00am – 10.30am

Syllabus: A wide ranging account on the elements of the spectral theory of linear operators with examples and canonical models from differential and integral equations.

- Bounded and unbounded linear operators, resolvent, spectrum, approximation and regularisation.
- Compact operators, algebraic and analytical properties of the family of compact operators, limit representation of compact operators, Hilbert-Schmidt operators, singular values
- Toy spectral theorems: Normal and selfadjoint operators, nonzero spectrum of compact normal operators, Riesz-Schauder theorem, the spectral theorem for compact normal operators
- Singular value decomposition, integral operators, Volterra operators, the Fredholm alternative, Fredholm operators, essential spectra, Weyl theorem, Full spectral theorem

Prerequisites: Advanced Linear Algebra, Measure and Integration, basic Functional Analysis, Fourier Series, basic Complex Analysis.

Supplementary (Advance) courses: Semester 2

Probabilistic Representation for Linear and Nonlinear PDE

David Siska & Lukasz Szpruch (20 hours, 15 credits)

Tuesdays 9:00am – 10:30am **(not available to SMSTC)**

Syllabus: Probabilistic interpretation of linear and non-linear PDEs in terms of Markov processes: show existence of solutions to some classes of linear and nonlinear PDEs and also to study their finer analytic properties using probabilistic tools

- Linear PDEs of elliptic and parabolic type and SDEs
- Semi-linear PDEs and BSDEs, non-linear Feynman-Kac formula
- Semi-linear PDEs and McKean–Vlasov SDEs, probabilistic representations to Boltzmann and Burger's equations
- Semi-linear PDEs and branching diffusions, probabilistic representation for KPP equation
- Linear SPDEs and SDE particle systems

Assessment: Active participation of students: leaving certain parts of the exposition to students as exercises, written solutions or presentations

Supplementary (Advance) courses: Semester 2

Variational Methods for PDEs *John M. Ball & Heiko Gimperlein*

Tuesdays 10.45am – 12.45pm

Format: 8 lectures in Week 1, 2 lectures per week, student lectures
Syllabus: Basic and advanced topics in calculus of variations and nonlinear PDEs, their applications and rigorous numerical approximation

- The 1D calculus of variations, Tonellis existence theorem, necessary & sufficient conditions for local minimizers, Lavrentiev phenomenon
- Introduction to the higher-dimensional calculus of variations: lower semicontinuity, notions of convexity, compensated compactness.
- Applications: nonlinear elliptic PDEs and the obstacle problem, equations of continuum mechanics, minimal surfaces.
- Numerical discretisation: Finite element approximation of nonlinear PDEs, obstacle problem and harmonic maps.
- Saddle point problems: Palais-Smale condition, Pohozaev's non-existence result, harmonic maps of Riemann surfaces, Convex integration and non-uniqueness of weak solutions of low regularity.

Student talks: Interested students will give a 60-minute lecture
Prerequisites: Previous course in either PDEs or their numerical analysis

Supplementary (Advance) courses: Semester 2

Two-dimensional statistical hydrodynamics, *Tadahiro Oh*

Thursdays 11.00 am – 12.50 pm **(not available to SMSTC)**

Syllabus: Evolution equations describing the motion of fluids, in particular from the statistical point of view, the randomly forced Navier-Stokes equation (NSE), the Euler equation as the limiting equation in the fluctuation-dissipation limit of the stochastic NSE.

- Deterministic well-posedness theory of NSE (namely, existence, uniqueness, and stability under perturbation of solutions).
- Stochastic NSE with additive stochastic forcing (uniqueness and ergodicity of the corresponding invariant measure)
- Construct an invariant measure for the Euler equation

Assessment: Small assignments, filling in details in lecture notes
S. Kuksin, *Randomly forced nonlinear PDEs and statistical hydrodynamics in 2 space dimensions*, Zurich Lectures in Advanced Mathematics. European Mathematical Society 2006