Unexpected connections

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Keep yourself open

and don't neglect your larger self

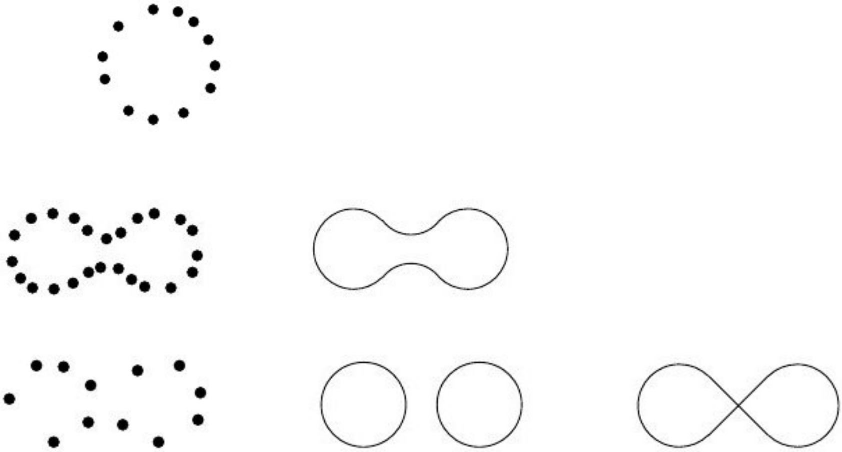
Applied maths

$\neq$ *applied differential equations*

$\neq$ *differential equations applied to physical problems*

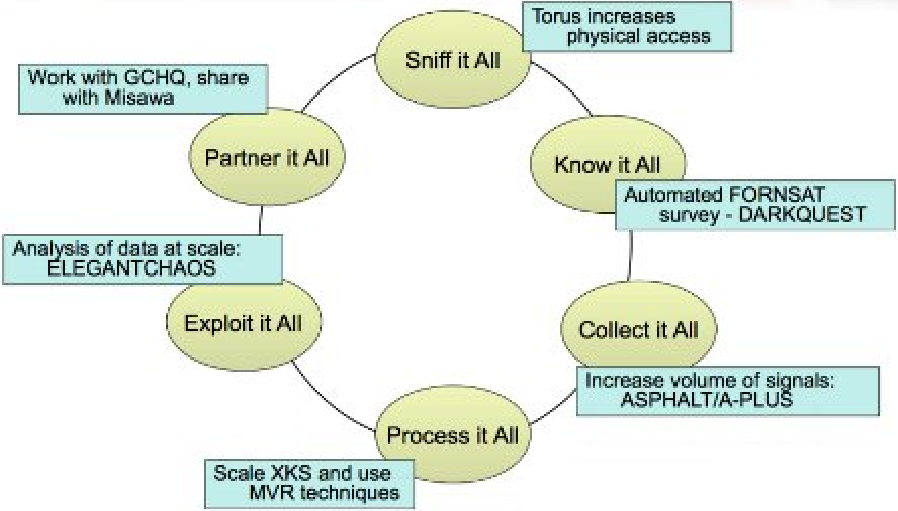
Algebraic topology of data sets

"What's this?" I ask:



[extracted from article by Vin de Silva]

New Collection Posture



DNA knotting

From the website of Dorothy Buck (Imperial):

RESEARCH INTERESTS

BIOMATHEMATICS:

- DNA-protein Interactions
- Site-specific recombination
- Mechanism of type-2 Topoisomerases
- Integron Integrases
- Mechanisms of Antibiotic Resistance

TOPOLOGY

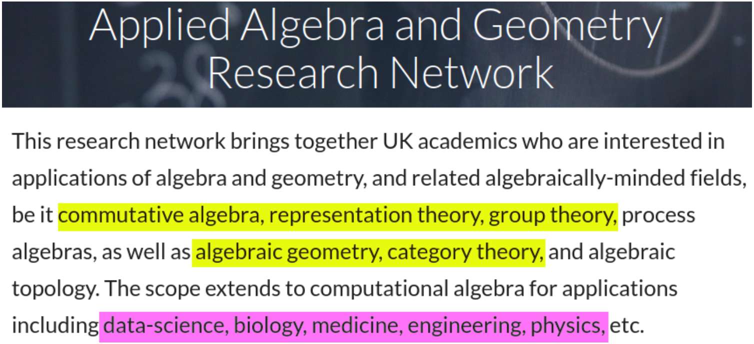
- Three-Manifolds
- Knot theory
- Dehn surgery
- Tangles
- Unknotting Number

FIGURE 1. In these examples the recombinase complex B meets the substrate in the two crossover sites (highlighted in black).

[source: Buck and Flapan]

A few other applications of mathematics

- Order theory, category theory and classical logic have all been used for the modelling and specification of concurrent systems.
- Topological data analysis, founded on the theory of persistent homology, discovered a new subtype of breast cancer with a 100% survival rate. [link]
- Many new applications of ' "pure" ' algebra are underway. . .



Applied Algebra and Geometry Research Network

This research network brings together UK academics who are interested in applications of algebra and geometry, and related algebraically-minded fields, be it commutative algebra, representation theory, group theory, process algebras, as well as algebraic geometry, category theory, and algebraic topology. The scope extends to computational algebra for applications including data-science, biology, medicine, engineering, physics, etc.

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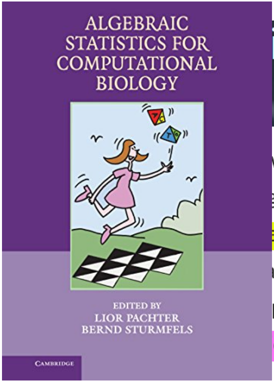

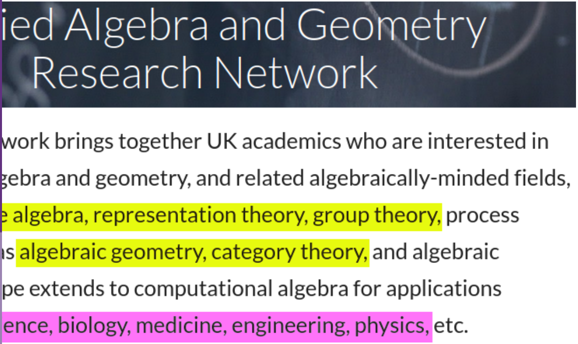

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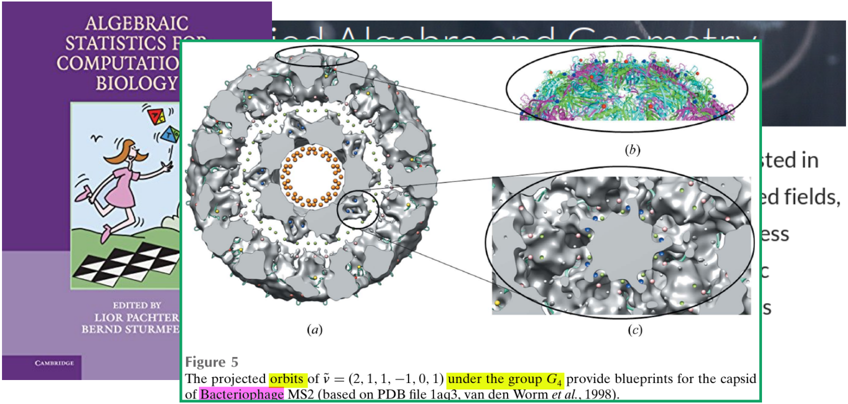

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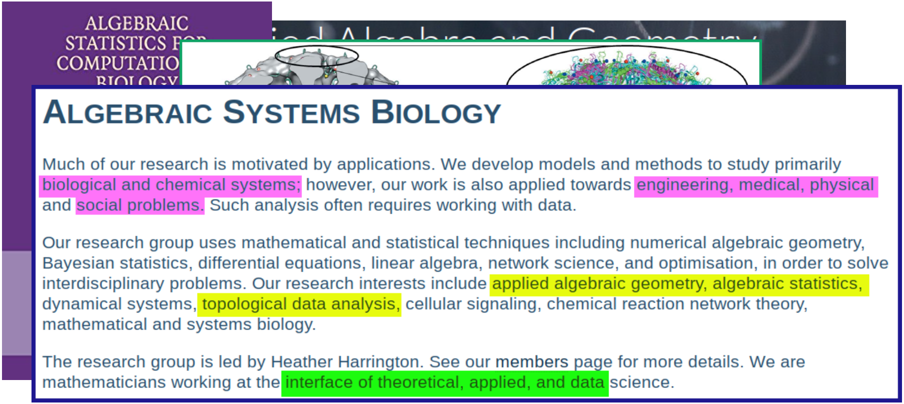


ALGEBRAIC SYSTEMS BIOLOGY

Much of our research is motivated by applications. We develop models and methods to study primarily biological and chemical systems; however, our work is also applied towards engineering, medical, physical and social problems. Such analysis often requires working with data.

Our research group uses mathematical and statistical techniques including numerical algebraic geometry, Bayesian statistics, differential equations, linear algebra, network science, and optimisation, in order to solve interdisciplinary problems. Our research interests include applied algebraic geometry, algebraic statistics, dynamical systems, topological data analysis, cellular signaling, chemical reaction network theory, mathematical and systems biology.

The research group is led by Heather Harrington. See our members page for more details. We are mathematicians working at the interface of theoretical, applied, and data science.

Applied maths
can help
pure maths

Random matrices and the Riemann zeta function

The year: 1972.

The scene: Afternoon tea at the Institute for Advanced Study, Princeton.

Freeman Dyson, dapper British physicist: 'So tell me, Montgomery, what have you been up to?'

Hugh Montgomery, boyish American mathematician: 'Well, lately I've been looking into the distribution of the zeros of the Riemann zeta function.'

Dyson: 'Yes? And?'

Montgomery: 'It seems the two-point correlations go as...' *(turning to write on a nearby blackboard)*

Dyson: Extraordinary! Do you realize that's the pair-correlation function for the eigenvalues of a random Hermitian matrix? It's also a model of the energy levels in a heavy nucleus — say uranium 238.

[adapted from web page by Bob McLeod]

Random matrices and the Riemann zeta function

[source: Bob McLeod]

Two more examples of applied helping pure

- The *Gruppenpest* (plague of groups) [link]

- My collaborator Mark Meckes: [link]

We end this section by considering a quantity related to magnitude which is in some ways better behaved. For a compact (not necessarily positive definite) metric space A, the **maximum diversity** of A is

$$|A|_+ = \sup_{\mu \in P(A)} \left(\int \int e^{-d(a,b)} \, d\mu(a) \, d\mu(b) \right)^{-1}, \tag{4.3}$$

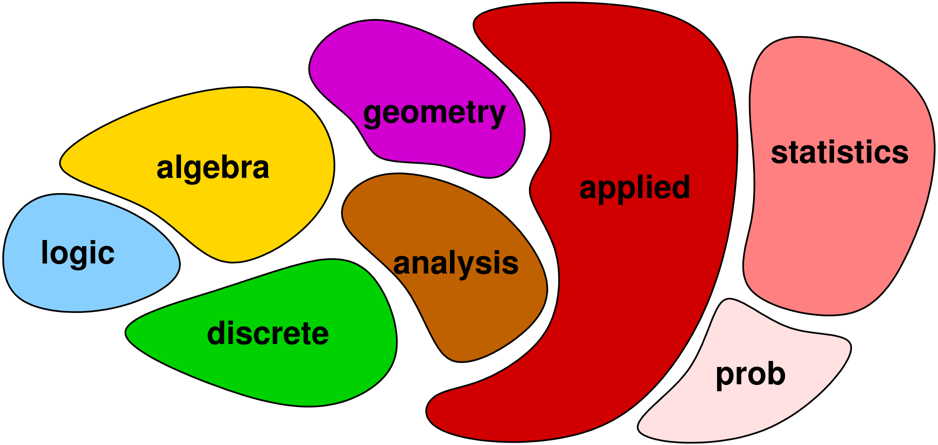
where $P(A)$ denotes the space of Borel probability measures on A. By renormalization, this is simply what one obtains by restricting the supremum in (3.5) to positive measures; thus we trivially have

$$|A|_+ \leq |A| \tag{4.4}$$

for any compact PDMS A. The name stems from the following interpretation of the quantity

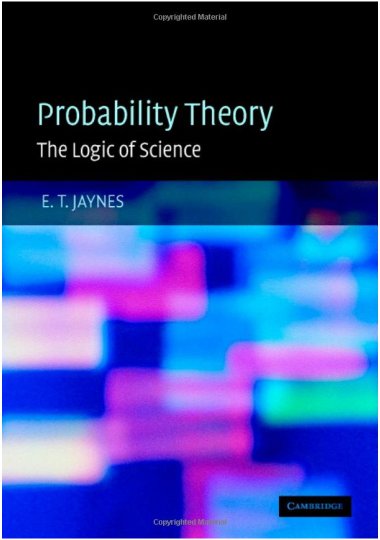
Prepare to rewire your brain

We all have a mental map of mathematics. . .



. . . but it can be misleading.

Statistical inference as a branch of logic



If A is true, then B becomes more plausible

B is true

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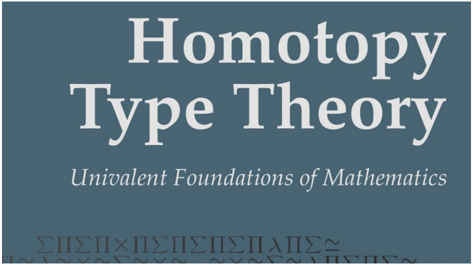
therefore, A becomes more plausible.

[Jaynes link]

Knowing unusual combinations of subjects gives you an advantage

Logic + topology + computer programming



Peter Aczel	Eric Finster	Alvaro Pelayo
Benedikt Ahrens	Daniel Grayson	Andrew Polonsky
Thorsten Altenkirch	Hugo Herbelin	Michael Shulman
Steve Awodey	André Joyal	Matthieu Sozeau
Bruno Barras	Dan Licata	Bas Spitters
Andrej Bauer	Peter Lumsdaine	Benno van den Berg
Yves Bertot	Assia Mahboubi	Vladimir Voevodsky
Marc Bezem	Per Martin-Löf	Michael Warren
Thierry Coquand	Sergey Melikhov	Noam Zeilberger

re were also the following students, whose participation was no less valuable.

Carlo Angiuli	Guillaume Brunerie	Egbert Rijke
Anthony Bordg	Chris Kapulkin	Kristina Sojakova

ddition, there were the following short- and long-term visitors, including student visit
se contributions to the Special Year were also essential.

Jeremy Avigad	Richard Garner	Nuo Li
Cyril Cohen	Georges Gonthier	Zhaohui Luo
Robert Constable	Thomas Hales	Michael Nahas
Pierre-Louis Curien	Robert Harper	Erik Palmgren
Peter Dybjer	Martin Hofmann	Emily Riehl
Martín Escardó	Pieter Hofstra	Dana Scott
Kuen-Bang Hou	Joachim Kock	Philip Scott
Nicola Gambino	Nicolai Kraus	Sergei Soloviev

Theorem	Status
$\pi_1(S^1)$	✔✔
$\pi_{k<n}(S^n)$	✔✔
long-exact-sequence of homotopy groups	✔✔
total space of Hopf fibration is S^3	✔
$\pi_2(S^2)$	✔✔
$\pi_3(S^2)$	✔
$\pi_n(S^n)$	✔✔
$\pi_4(S^3)$	✔
Freudenthal suspension theorem	✔✔
Blakers–Massey theorem	✔✔
Eilenberg–Mac Lane spaces $K(G, n)$	✔✔
van Kampen theorem	✔✔
covering spaces	✔✔
Whitehead's principle for n-types	✔✔

Table 8.2: Theorems from homotopy theory proved by hand (✔) and by computer (✔✔).

You can't learn everything yourself. . .

. . . but you need to know enough to be able to communicate with your collaborators

Expect the unexpected!