Black Hole

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- In this course we shall study about black holes from both classical and quantum perspectives.
- Let us begin by briefly discussing why I find it interesting to study Black Holes.

What is gravity

- Einstein's General Theory of Relativity (GTR) is a classical theory of gravity
- According to GTR, the curvature of the space-time is the source of gravitational force.
- The curvature of the space-time is mathematically encoded in the metric tensor: $g_{ab} \rightarrow$ the basic variable of GTR
- g_{ab} measures the distance between two infinitesimally separated points on the manifold of space-time.

What are Black Holes

- The dynamics of $g_{ab}(x)$ is governed by Einstein's equations.
- Einstein's equations are a set of coupled second order partial differential equations involving the components of $g_{ab}(x)$.
- Black Holes (BH) are singular solutions of Einstein's equations.
- In black hole space-time, the structure of $g_{ab}(x)$ is such that the space-time curvature ($\equiv$ the gravitational attraction) becomes infinite at some point(s).

Black hole and its infinite gravitational pull

- Now any effect that is experimentally observable cannot be strict infinity.
- The fact that existence of Black Holes predicts infinite gravitational pull at some point(s) in space and time indicates that
  1. either black hole solutions are just some mathematical constructions that can never occur in nature.
  2. or near this (these) singular point(s) the formulation of GTR breaks down.

The infinite gravitational pull (contd)

The possibilities:

1. either black hole solutions are just some mathematical constructions that can never occur in nature.
2. or near this (these) singular point(s) the formulation of GTR breaks down.

- Now we have enough observational evidence to rule out the possibility - 1.
- Black Holes do exist in nature and so we need to re examine GTR at least near the singular points to cure the problem of infinities.

One immediate ‘classical’ cure of this particular infinity

- It turns out that for most Black Hole solutions the singularity is ‘causally shielded’ from the outside world by a surface, called event horizon.
- It means that no event inside the event horizon could be the cause of any event outside it.
- In other words, though there might exist a point of ‘infinite gravitational pull’ behind the event horizon, it has zero effect on any of our experiments conducted outside the event horizon.
- This does solve the problem of immediate contradiction that arises from predicting infinite gravitational pull somewhere
- But this feels a bit unsatisfactory, it is not a ‘complete cure of the problem’ but just pushing the problem behind the event horizon.

Black hole thermodynamics $\to$ a cure in quantum gravity

It turns out that

- some components of Einstein's equations, evaluated on the horizon, could be rearranged and interpreted as Zeroth, First and Second law of thermodynamics

  provided we identify some geometric properties of BH with thermodynamic properties.
- For example,
  - mass of the black hole $\longrightarrow$ total energy.
  - surface gravity ($\sim$ the gravitational pull at the horizon) $\longrightarrow$ temperature.
  - The area of the horizon $\longrightarrow$ entropy.

Black hole thermodynamics $\rightarrow$ a cure in quantum gravity

- Now we know thermodynamics is a collection of emergent statistical properties which apply to only large systems with many constituent particles and configurations.
- However, in determining the black hole solutions in GTR we never need to use any statistics or concept of probabilities.
- The fact that we end up with a geometry satisfying thermodynamic laws indicates that black holes (or maybe the whole concept of space-time geometry) is an emergent phenomenon of a large number of quantum microstates.
- And the resolution of the classical infinities of black holes would come from an appropriate formulation of quantum gravity.

Why the study of black holes interesting

- To my knowledge, we do not yet have a truly satisfactory resolution of this problem of infinities in black holes and in the classical/quantum theories of gravity in general.
- And it turns out that the black hole solutions play the role of a theoretical laboratory in this context.
- We can use these solutions to test the present and future theories of gravity, their consistency and connection with other branches of physics like thermodynamics and so on.
- For example, black holes encode the break down classical theory of gravity (GTR).
  (Not by any contradictory experimental observation, but by generating a theoretical inconsistency of infinite gravitational force.
  This is why the black holes are 'theoretical laboratories'.)

This course - the plan

- It would be great to run this course in the most interactive way so that we all can learn together something that was not known to us before.
- I do have a plan of the course now but we might need to change depending on our collective interest at some later point.
- So please let me know if you have any concern / comment related to the content or other aspects of the course at any point of time.
- At the moment I have planned the course in two parts.

This course - the first part

- In the first part we shall discuss black holes from the perspective classical physics, (i.e.,GTR) that is quite well understood.
- We shall discuss many different stationary black hole solutions, the causal structure of the space-time and the event horizon.

  Prerequisite: Though we shall briefly review but familiarity with the concept of space-time geometry, curvature tensor and Einstein's equations would be useful here.
- Next we shall study the emergent thermodynamic properties of the stationary black hole.

  prerequisite: Some exposure to thermodynamics and its interpretation in terms of statistical physics will probably allow a better appreciation of the results we shall learn in this part.
- As I mentioned before, these concepts are well understood by now and there are many excellent books and lecture notes on these topics, which we shall follow.

This course - the second part

- In this part we shall attempt to learn topics in the areas of active research.
- As a result, this part of the course will be more fluid in nature and will depend on how the first part goes as well as your feedback at the end of the first part.
- We shall first attempt to learn techniques to construct several approximate black hole solutions that are evolving with time.
- Then we shall delve into a quantum or semi-classical (a first quantum correction to classical GTR) description of black holes leading to Hawking radiation.
- We shall learn that black holes can radiate energy like a black-body and then can eventually evaporate.

pre-requisite: An introduction to quantum mechanics, particularly to the concept of particle 'creation' and 'annihilation' operators is needed.

This course - the second part

- The radiation and the final evaporation of black hole lead to the paradox related to the 'information loss'.
- We shall first try to understand the paradox in different forms.
- Then we shall study the several attempts to its resolution.
- At the very end I hope to understand the recent idea of emergent gravity from entanglement entropy (a very active area of research at the moment).

That's all for now from my side.
Thank you for listening.