# Quantum Computing: Introduction (to an Introductory course)

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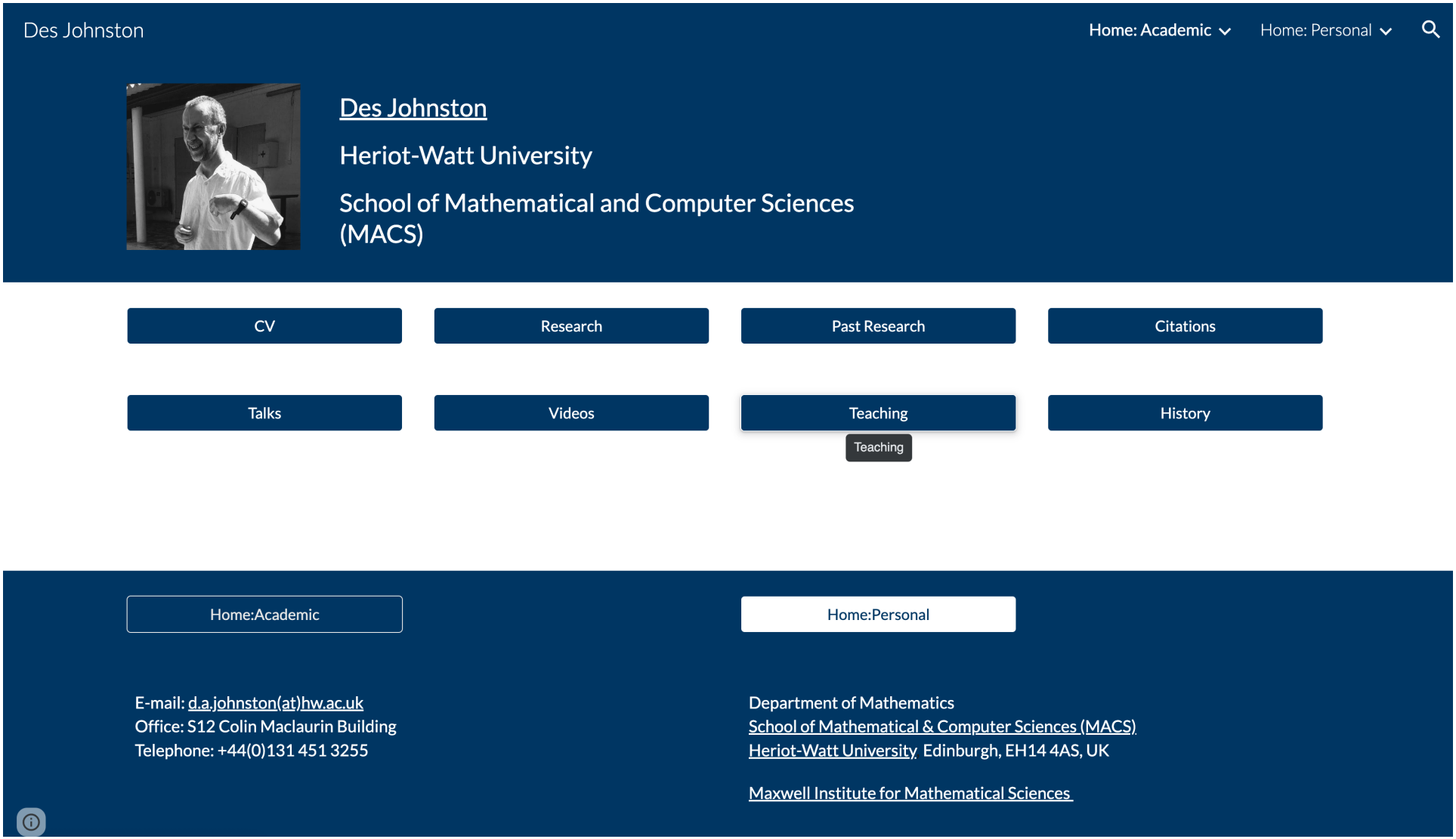
# Who am I?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# Why you shouldn't do this course

- I'm an amateur, so it's done (much) better elsewhere

  https://qiskit.org
- It'll annoy you if you have a physics background
- It'll annoy you if you have a maths background
- It'll annoy you if you have a computer science background

# Physics Annoyances

- I make a big deal out of familiar notation
  ($\langle 0| , |0\rangle , \langle 0|0\rangle , \langle 0|A|0\rangle \ldots$)
- I'll say very little about noise and quantum error correction
- I ignore physical implementation

# Maths Annoyances

- I make a big deal out of weird physics notation ($\langle 0| \, , |0\rangle \, , \langle 0|0\rangle \, , \langle 0|A|0\rangle \ldots$)
- Degree of rigour is zero
- Corner cases ignored, awkwardness swept under carpet

# Computer Science Annoyances

- Only a token effort is made to count the computational cost of various algorithms
- Questions of quantum advantage largely ignored
- Corner cases ignored, awkwardness swept under carpet (again)

# If this hasn't put you off - What are you letting yourselves in for?

- Quantum mechanics - rules of the game, superposition, entanglement
- Quantum circuit model of quantum computing
- Party tricks - dense coding, teleportation.....
- Quantum advantage - Deutsch's algorithm
- Finding a needle in a haystack - Grover's algorithm
- Breaking the code - Shor's algorithm
- More stuff

# Our Mission, should we choose to accept it...

Understanding how to "read" quantum circuits and bra/ket notation, such as....

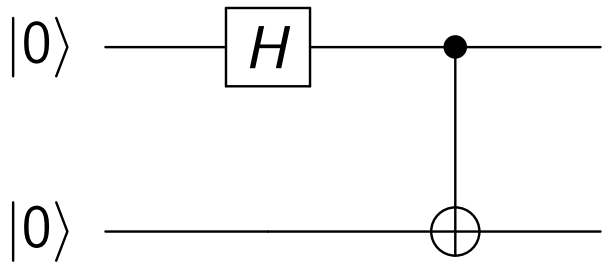



Figure: Quantum circuit for preparing the Bell state $|\psi^{00}\rangle$

## Step 1

$$H\otimes I\,|00\rangle = (H\,|0\rangle)\otimes(I\,|0\rangle) = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)\otimes|0\rangle = \frac{1}{\sqrt{2}}(|00\rangle+|10\rangle).$$

## Step 2

$$\mathrm{CNOT}_{12}\left(\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)\right) = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = |\psi^{00}\rangle$$

# Do Quantum party tricks

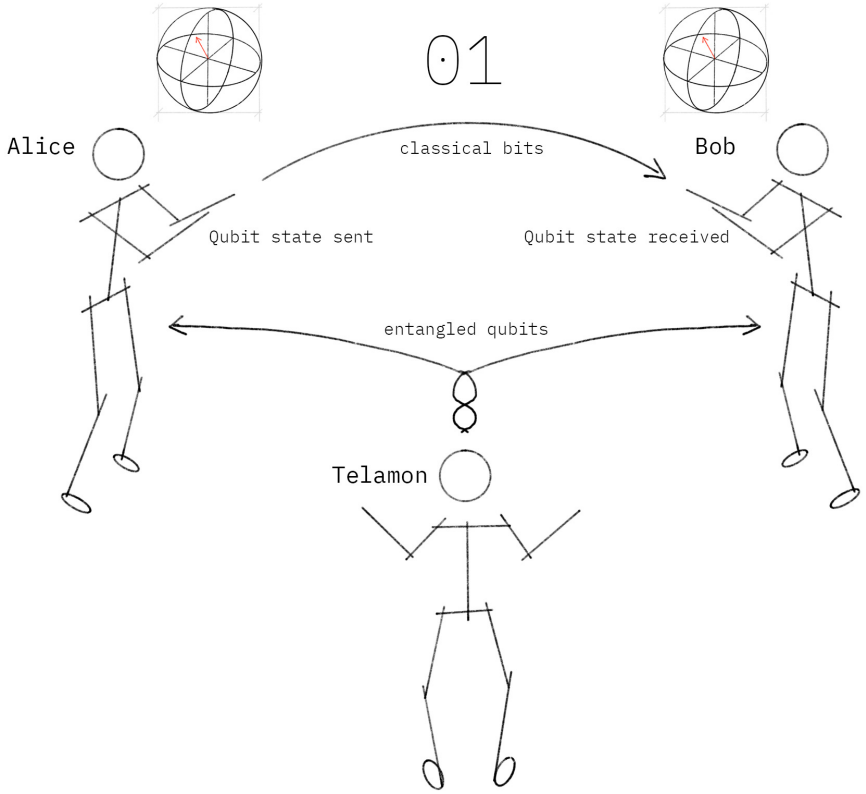
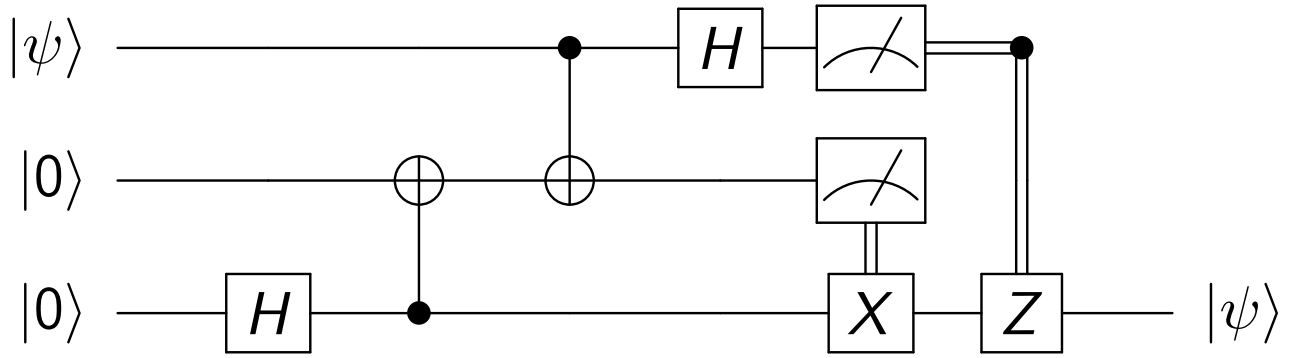



Figure: Alice teleports a state to Bob

# Teleportation - Circuit



# Quantum Advantage - Deutsch

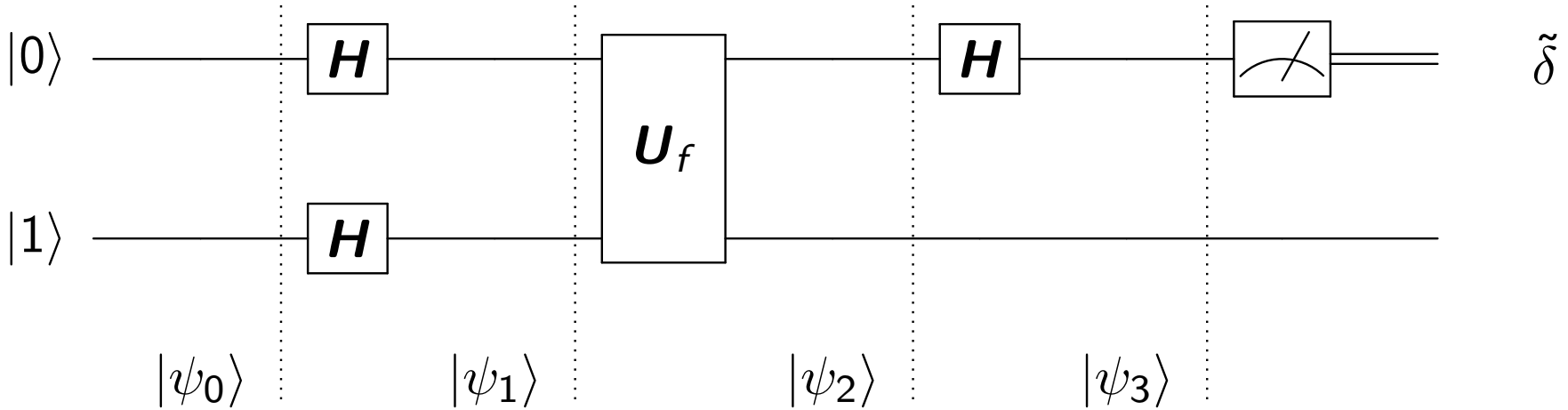
Distinguish a constant

$$f(0) = f(1)$$

from a balanced

$$f(0) \neq f(1)$$

binary function with one query

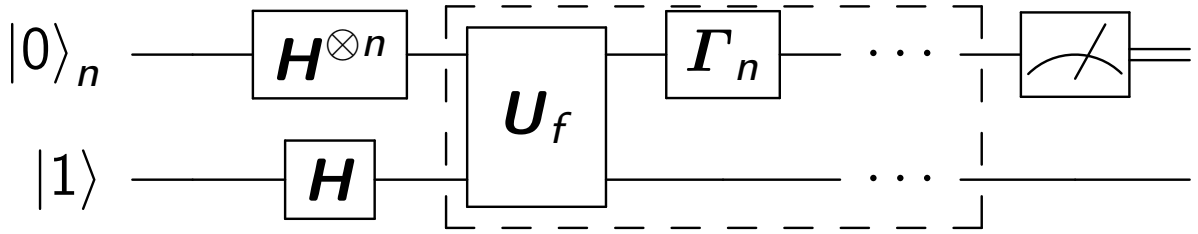


# Finding a Needle in Haystack - Grover

- Unstructured database with $2^n$ elements, identified by the indices $0, \ldots, 2^n - 1$
- We want to find one marked element that satisfies a certain property
- We can test for this once we have the element
- Worst case classically - find the marked element last, requiring $\mathcal{O}(2^n)$ queries

Grover's algorithm makes only $\mathcal{O}(\sqrt{2^n})$ queries to the database, square root speedup

# Grover - circuit




Repeat $\mathcal{O}(\sqrt{2^n})$ times

In $U_f$

$$f(j) = \begin{cases} 1 & \text{if } j = j_0 \\ 0 & \text{otherwise} \end{cases}$$

# We might even factor 15 using Shor's algorithm

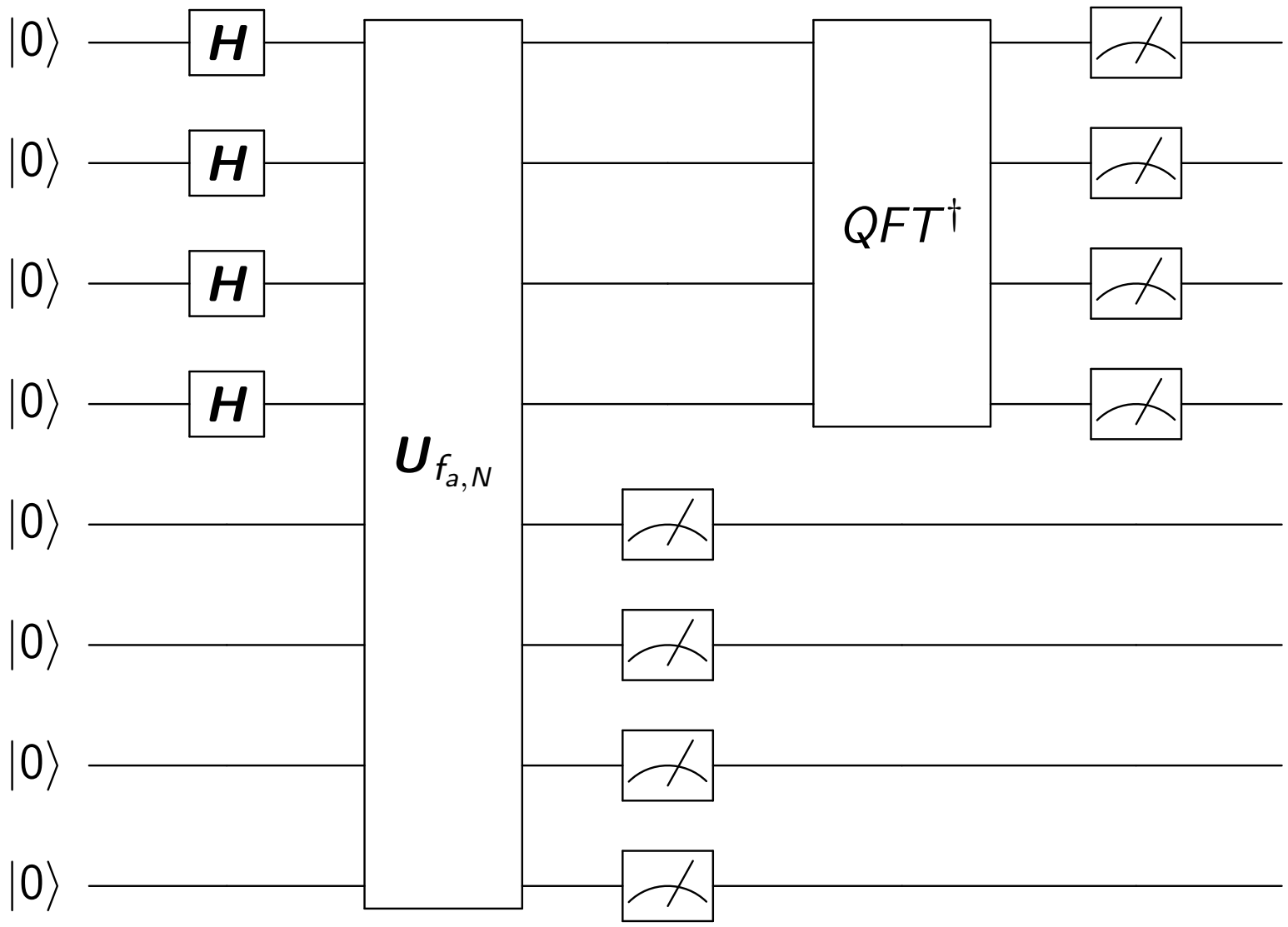
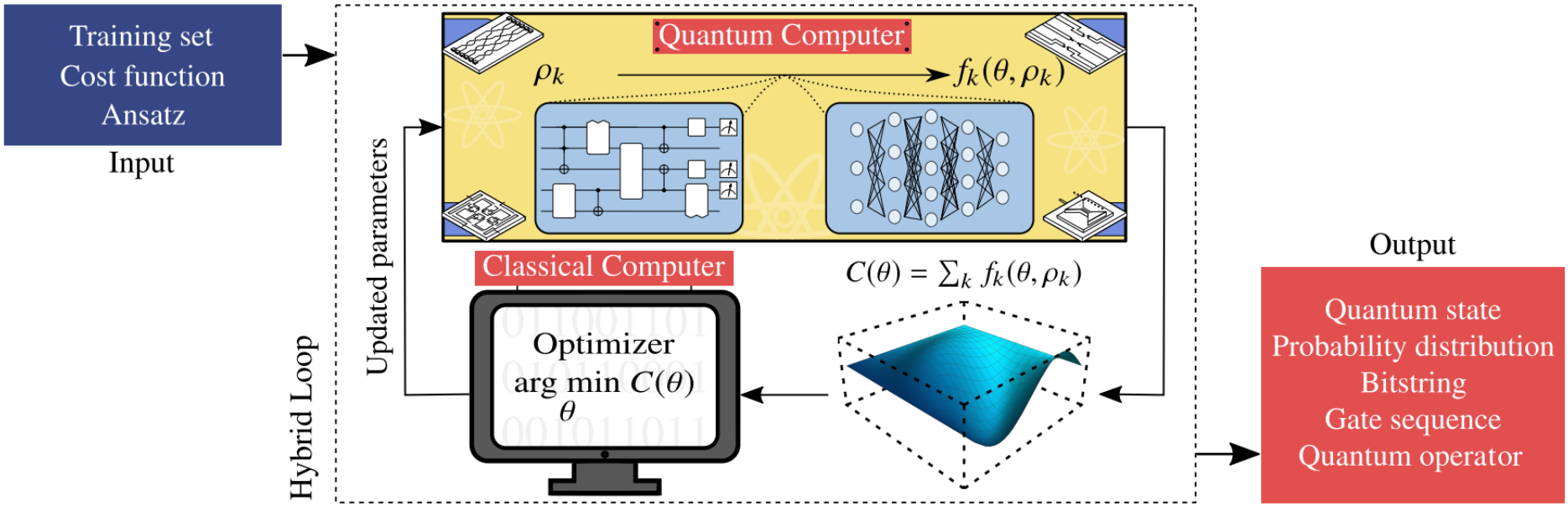



Figure: Factor 15 (5,3)

# Stuff folk actually do - Variational Quantum Algorithms



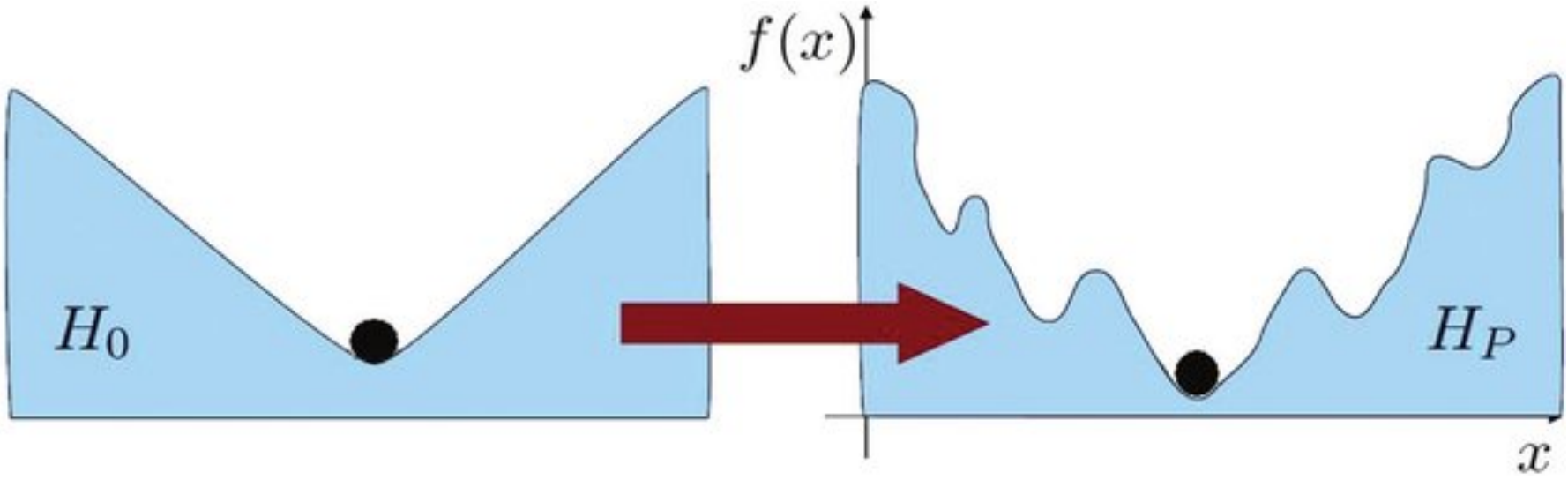

- Find eigenvalues: Variational Quantum Eigensolver (VQE)
  $\Longrightarrow$ Simulate molecules in computational chemistry
- **QAOA:** a quantum combinatorial optimisation algorithm

# An Alternative Paradigm - Adiabatic Quantum Computation

Useful for problems for which the solution corresponds to the (ground) state of a Hamiltonian $\hat{H}_p$.

Idea: Design a schedule $A(s)$ to move from $\hat{H}_0 = \hat{H}(0)$ to $\hat{H}_p = \hat{H}(\tau)$ in a way that $\psi(\tau) \approx \psi^{(j)}(\tau)$

$$\hat{H}(s) = (1 - A(s))\hat{H}_0 + A(s)\hat{H}_p$$




[H. Krovi, 2016]

# Useful Resources I

Online:

Check out Qiskit (which does a **much** better job of explaining things than I ever will)

https://qiskit.org

Play with simulators, online and downloadable, and even real (but not very big) quantum computers.

Downloadable version runs well in a Jupyter notebook.

# Useful resources II

ArXiv:

*Quantum computing from a mathematical perspective: a description of the quantum circuit model* by J. Ossorio-Castill and José M. Tornero

https://arxiv.org/abs/1810.08277

*Quantum Algorithm Implementations for Beginners* by 20 million authors

https://arxiv.org/abs/1804.03719

(which I will both shamelessly plagiarize, along with Qiskit)

# Useful resources III

Books:

*Quantum Computation and Quantum Information* by Michael A. Nielsen and Isaac L. Chuang

*Quantum Computer Science: An Introduction* by N. David Mermin