SMSTC, Structure and Symmetry

Ana G. Lecuona, Glasgow
ana.lecuona@glasgow.ac.uk

Structure & Symmetry

Wikipedia tells us:

- **Algebra** (from Arabic "al-jabr", literally meaning "reunion of broken parts") is the study of mathematical symbols and the rules for manipulating these symbols.
- **Geometry** (from the Ancient Greek: geo- "earth", -metron "measuremen") is a branch of mathematics concerned with questions of shape, size, relative position of figures, and the properties of space.
- **Topology** (from the Greek topos, place, and logos, study) is concerned with the properties of a geometric object that are preserved under continuous deformations, such as stretching, twisting, crumpling and bending, but not tearing or gluing.

Theme overview

Semester 1

★ **Groups, Rings and Modules**

- Collin Bleak, Saint Andrews
  cb211@st-andrews.ac.uk
- Tom Coleman, Saint Andrews
  tdhc@st-andrews.ac.uk
- Carlo Pagano, Glasgow
  carlo.pagano@glasgow.ac.uk

★ **Algebraic Topology**

- Christian Bönicke, Glasgow
  christian.bonicke@glasgow.ac.uk
- Dinakar Muthiah, Glasgow
  dinakar.muthiah@glasgow.ac.uk

Theme overview

Semester 2

★ **Algebras and Representation Theory**

- Carlo Pagano, Glasgow
  carlo.pagano@glasgow.ac.uk
- Rekha Biswal, Edinburgh
  rbiswal@ed.ac.uk

★ **Manifolds**

- Fei Xie, Edinburgh
  fei.xie@ed.ac.uk
- Jelle Hartong, Edinburgh
  jelle.hartong@ed.ac.uk

Prerequisites

- ★ **Groups, Rings and Modules**
  - Basic linear algebra and basic algebra concepts.
    - Definitions and examples of groups, rings and fields.
  - Basic notions of group theory.
    - Lagrange's theorem, normal subgroups and factor groups.
- ★ **Algebraic Topology**
  - A course in metric spaces or topological spaces.
  - A course in group theory.
    - Group actions.
    - Finitely generated abelian groups.

Prerequisites

- ★ **Algebras and Representation Theory**
  - The notion of a module and related concepts.
  - Basics on noetherian and artinian modules.
  - Some commutative algebra.
- ★ **Manifolds**
  - Standard calculus courses.
    - Green's theorem.
  - Basic courses in linear algebra.
    - Abstract vector space.

Groups, Rings and Modules

- Groups.
  1. Simple groups, Jordan-Holder Theorem, (semi)direct products.
  2. Permutation representations and group actions.
  3. Sylow Thorems and applications.
  4. Abelian, soluble and nilpotent groups.
  5. Free groups and presentations.
- Commutative rings.
  1. Modules: introduction.
  2. Chain conditions and Hilbert's basis theorem.
  3. Fields and numbers.
  4. Affine algebraic geometry.
  5. Hilbert's Nullstellensatz.

Representation Theory

- Noncommutative rings.
  - Finitely generated modules over principal ideal domains.
  - The Artin-Wedderburn Theorem.
- Representation Theory.
  - Representations and characters.
  - Orthogonality relations.
  - Induced representations.
  - Computing character tables.

Algebraic Topology

(1) Basic examples and constructions of topological spaces.
(2) Manifolds, basic homotopy theory and homotopy groups.
(3) Cofibrations, cell attachments and CW-complexes.
(4) Cellular approximation and relative homotopy groups.
(5) Fibre bundles, fibrations and the Hopf map.
(6) An introduction to homology.
(7) Homotopy invariance, exactness and excision.
(8) Computations and applications of homology.
(9) An introduction to cohomology.

Manifolds

(1) Implicit Function and Sard's Theorems, abstract manifolds.
(2) Tangent vectors and the tangent bundle, vector bundless.
(3) Vector fields and flows, Lie derivative, the Frobenius Theorem.
(4) Differential forms, Stokes' Theorem and Poincare duality.
(5) Riemannian metrics, connections, the Levi-Civita connection.
(6) Geodesics, the exponential map.
(7) Curvature and integrability, Riemannian curvature.
(8) Gauss Formula and the Theorema Egregium.
(9) Euler characteristic, the Gauss-Bonnet Theorem for surfaces.

Enjoy the Theme!