Mathematical Models

Thursdays 15.30-17.30

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Overview

Mathematics can be used to understand and quantify real-world phenomena

The idea is to form a mathematical model

– a system of equations which describe these phenomena

Once the model has been validated, it can be used to make predictions of how the system will behave

Perspective

This course will:

1. Focus on applications
2. Explain the origin and construction of mathematical models
3. Derive these models as ODEs and PDEs
4. Focus on constructing solutions to particular problems
5. Examine the practical implications of the model predictions

This is not an applied analysis or methods course!

Where mathematical models come from

1. Basic physical laws
2. Observations of the system behaviour
3. Intuition of what might give “correct” behaviour

Problem driven: <u>not</u> maths driven

Breakdown of lectures

MODELS 1: Continuum modelling

- **Rational mechanics**

Penny Davies, University of Strathclyde

- **Fluid mechanics**

Peter Stewart & Nick Hill, University of Glasgow

Iain Stewart, University of Dundee

Although some of you may have taken courses in continuum mechanics before - this will include lots of ideas not covered in UG lectures (eg lubrication theory, non-Newtonian fluids etc)

Breakdown of lectures

MODELS 1: Continuum modelling

- **Rational mechanics**

Introduction to tensors
Kinematics
Balance laws
Cauchy's theorem of stress
Constitutive laws

Breakdown of lectures

MODELS 1: Continuum modelling

- **Fluid mechanics**

General concepts
Surface water waves
Boundary layer theory
Lubrication theory
Classical aerofoil theory

Continuum theory of non-Newtonian fluids, including Ostwalde-de Waele, Bingham, Herschel-Bulkley and comparisons with Newtonian fluids.

Breakdown of lectures

MODELS 2: Application of models to real-world problems

- **Mathematical Physiology**

Peter Stewart & Radostin Simitev (University of Glasgow), Mark Chaplain (University of St Andrews)

Airflow in the lungs, blood flow in arteries and veins
Modelling of electrophysiology
Mathematical modelling of cancer

Breakdown of lectures

MODELS 2: Application of models to real-world problems

- **Mathematical Biology**

Rachel Norman & Andy Hoyle (University of Stirling),
Dougie Spiers (University of Strathclyde),
Frits Veerman & Tibor Antal (University of Edinburgh)

Population modelling
Evolution
Age-structured models
Patterns and waves in Turing models
Branching processes in biology

Prerequisites

- Basic linear algebra
- Vector calculus
- Introductory courses on ODEs and PDEs

No prior experience of continuum mechanics or mathematical modelling is required.

Come along and learn some new skills!

Assessment

- One assessment on each module divided into two equally weighted parts:

MODELS 1 (1) Continuum mechanics
(2) Fluid mechanics

MODELS 2 (1) Mathematical physiology
(2) Mathematical biology