SMSTC Algebra I & II

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Perth 29/09/2016

What is Algebra?

- Traditionally: Theory of polynomials and solving equations.
- 19th, 20th, 21st Centuries: Theory of various abstract algebraic structures.
- Algebraic structure: A set with some operations defined on it.

Areas of Algebra

Division according to the number of operations and their properties.

- Classical structures:
  - Groups, rings, fields
  - Linear spaces, modules
  - Algebras, Lie algebras
- 'Modern' structures:
  - Lattices, semigroups, general/universal algebras, boolean algebras, quasigroups, semirings, Hopf algebras, vertex operator algebras, differential graded algebras,....

Content

In this course, we will concentrate on the more classical structures

- Part 1: Groups
- Part 2: Commutative Rings
- Part 3: Non-commutative rings.
- Part 3: Representation theory

Unifying theme

Emphasis on interplay between different areas and structures.

For example: actions.

- Part 1: Groups: Groups acting on sets
- Parts 2 and 3: Rings: Rings acting on modules, matrix rings
- Part 4: Representation theory: groups (and group rings) acting on vector spaces

Groups (5 lectures)

- Topics:
  - Simple groups, Jordan–Hölder theorem, direct and semidirect products
  - Permutation representations and group actions
  - Sylow Theorems and applications
  - Abelian, soluble and nilpotent groups
  - Free groups and presentations
- Lecturers:
  - **Colva Roney-Dougal** (St. Andrews)
  - **Martyn Quick** (St. Andrews)

Sample result

Theorem

The following are (up to isomorphism) all the groups of order less than 16*:*

Order	*Groups*
1	1
2	C_2
3	C_3
4	C_4, K_4
5	C_5
6	C_6, $D_3 = S_3$
7	C_7
8	D_4, Q_8, C_8, $C_4 \times C_2$, $C_2 \times C_2 \times C_2$,

Order	*Groups*
9	C_9, $C_3 \times C_3$
10	C_{10}, D_5
11	C_{11}
12	D_6, A_4, T, C_{12} $C_2 \times C_2 \times C_3$
13	C_{13}
14	C_{14}, D_7
15	C_{15}

Commutative rings (5 lectures)

- Topics:
  - Modules: introduction
  - Chain conditions and Hilbert's basis theorem
  - Fields and numbers
  - Affine algebraic geometry and Hilbert's Nullstellensatz
- Lecturers:
  - **Ulrich Krähmer** (Glasgow)

Noncommutative rings (5 lectures)

- Topics:
  - Finitely generated modules over principal ideal domains and applications
  - The Artin–Wedderburn theorem
- Lecturers:
  - **Chunyi Li** (Edinburgh)

Representation theory (5 lectures)

- Topics:
  - Representations and characters
  - Orthogonality relations
  - Induced representations
  - Computing character tables
  - Applications
- Lecturer:
  - **Ben Martin** (Aberdeen)

Sample result

Theorem (Burnside)

Every group of order $p^a q^b$ where p and q are primes is soluble.

Prerequisites

You should be familiar and comfortable with:

- Basic linear algebra
- Definitions and examples of groups, rings, fields
- Basic algebra concepts such as homomorphisms
- Basic notions of group theory: permutations, symmetric groups, Lagrange's theorem, normal subgroups and factor groups

If you want to join in 2nd term you should know:

- The notion of a module and related concepts.
- Basics on Noetherian and Artinian modules.
- Some commutative algebra, in particular the notion of a principal ideal domain.

Other Details

- Lecture time: Mondays 1pm–3pm
- First lecture: next Monday, 3 Oct, from St Andrews
- Please read Sections 1.1 and 1.2 of the notes before the first lecture - the material in Section 1.1 is revision and will not be covered in lectures.
- Tutorial and IT support: demand that this is arranged locally
- Assessment: continuous; four take-home sets of problems (two in each term).