

# Groups, Rings and Modules and Algebras and Representation Theory

Iain Gordon  
igordon@ed.ac.uk

School of Mathematics, University of Edinburgh

Perth 5 Oct 2017



# Iain Gordon (Stream leader)

- ▶ University: Edinburgh
- ▶ Research interests:  
Geometric Representation  
Theory and applications  
to Algebraic Geometry  
and to Combinatorics

- ▶ Web page:

<http://www.maths.ed.ac.uk/~igordon/>

- ▶ E-mail:

[igordon@ed.ac.uk](mailto:igordon@ed.ac.uk)



# Colva Roney-Dougal

- ▶ University: St Andrews
- ▶ Research interests:  
symmetry and inference,  
permutation groups,  
matrix groups, constraint  
satisfaction
- ▶ Web page:  
`www-groups.mcs.st-and.ac.uk`  
`/~colva/`
- ▶ E-mail:  
`colva@mcs.st-and.ac.uk`



# Martyn Quick

- ▶ University: St Andrews
- ▶ Research interests: group theory, finite and infinite
- ▶ Web page:  
`www-groups.mcs.st-and.ac.uk/~martyn/index.html`
- ▶ E-mail:  
`mq3@st-andrews.ac.uk`



# Ellen Henke

- ▶ University: Aberdeen
- ▶ Research interests: Finite group theory, particularly fusion systems
- ▶ Web page:  
[www.maths.abdn.ac.uk/ncs/people/profiles/ellen.henke](http://www.maths.abdn.ac.uk/ncs/people/profiles/ellen.henke)
- ▶ E-mail:  
[ellen.henke@abdn.ac.uk](mailto:ellen.henke@abdn.ac.uk)



# Greg Stevenson

- ▶ University: Glasgow
- ▶ Research interests: tensor and triangulated categories
- ▶ Web page:  
`www.maths.gla.ac.uk/~gstevenson/`
- ▶ E-mail:  
`Gregory.Stevenson@glasgow.ac.uk`



# Charlie Strickland-Constable

- ▶ University: Edinburgh
- ▶ Research interests:  
generalised geometries  
and mathematical physics
- ▶ Web page:  
[http://www.maths.ed.ac.uk/  
school-of-mathematics/people?person=590](http://www.maths.ed.ac.uk/school-of-mathematics/people?person=590)
- ▶ E-mail:  
`charles.strickland-constable@ed.ac.uk`



# Laura Ciobanu

- ▶ University: Heriot-Watt
- ▶ Research interests:  
Combinatorial, geometric  
and algorithmic group  
theory. Combinatorics  
and theoretical computer  
science.
- ▶ Web page:  
<http://www.macs.hw.ac.uk/~lc45/>
- ▶ E-mail:  
[l.ciobanu@hw.ac.uk](mailto:l.ciobanu@hw.ac.uk)





# What is Algebra?

- ▶ Traditionally: Theory of polynomials and solving equations.
- ▶ 19th, 20th Centuries: Theory of various abstract **algebraic structures**.
- ▶ Algebraic structure: A set with some operations defined on it.

# Areas of Algebra

Division according to the number of operations and their properties.

- ▶ Classical structures:
  - ▶ Groups, rings, fields
  - ▶ Linear spaces, modules
  - ▶ Algebras, Lie algebras
- ▶ 'Modern' structures:
  - ▶ Lattices, semigroups, general/universal algebras, boolean algebras, quasigroups, semirings, Hopf algebras, vertex operator algebras, differential graded algebras, . . . .

# Content

In this course we will concentrate on the classical structures:

## Groups, Rings and Modules

- ▶ Part 1: Groups (5 lectures)
- ▶ Part 2: Commutative Rings (5 lectures)

## Algebras and Representation Theory

- ▶ Part 1: Noncommutative Algebra (5 lectures)
- ▶ Part 2: Representation theory (5 lectures)

# Groups (5 lectures)

- ▶ Topics:
  - ▶ Simple groups, Jordan–Hölder theorem, direct and semidirect products
  - ▶ Permutation representations and group actions
  - ▶ Sylow Theorems and applications
  - ▶ Abelian, soluble and nilpotent groups
  - ▶ Free groups and presentations
- ▶ Lecturers:



# Commutative rings (5 lectures)

- ▶ Topics:
  - ▶ Modules: introduction
  - ▶ Chain conditions and Hilbert's basis theorem
  - ▶ Fields and numbers
  - ▶ Affine algebraic geometry and Hilbert's Nullstellensatz
- ▶ Lecturers:



# Noncommutative rings (5 lectures)

- ▶ Topics:
  - ▶ Finitely generated modules over principal ideal domains and applications
  - ▶ The Artin–Wedderburn theorem
- ▶ Lecturers:



# Representation theory (5 lectures)

- ▶ Topics:
  - ▶ Representations and characters
  - ▶ Orthogonality relations
  - ▶ Induced representations
  - ▶ Computing character tables
  - ▶ Applications
- ▶ Lecturer:



## Prerequisites

You should be familiar and comfortable with:

- ▶ Basic linear algebra
- ▶ Definitions and examples of groups, rings, fields
- ▶ Basic algebra concepts such as homomorphisms
- ▶ Basic notions of group theory: permutations, symmetric groups, Lagrange's theorem, normal subgroups and factor groups

If you want to join in 2nd term you should know:

- ▶ The notion of a module and related concepts.
- ▶ Basics on Noetherian and Artinian modules.
- ▶ Some commutative algebra, in particular the notion of a principal ideal domain.



## Other Details

- ▶ Lecture time: Mondays 1pm–3pm
- ▶ First lecture: next Monday, 9 Oct, from St Andrews
- ▶ Tutorial and IT support: this is arranged locally
- ▶ Assessment: continuous; four take-home sets of problems (two in each term).