SMSTC: Applications of Mathematics (2018–2019)

Overview. The *Applications of Mathematics* theme covers some of the major topics in applied mathematics. We will look at how models are constructed and analysed in two important areas: continuum mechanics and mathematical biology. In parallel with this, we will look at two complementary sets of mathematical tools for analysing problems to which exact solutions are unavailable: analytical approximations and numerical approximations. Although the models and the methods we cover are 'classical' applied mathematics, they have applications across a wide range of contemporary research areas.

Analytical and Asymptotic Methods (Semester 1)

(i) Asymptotic methods for differential equations, including the methods of multiple scales and matched asymptotics (lectures 1 to 5; Dr David Pritchard, Prof. Alan Hood and Dr Antonia Wilmot-Smith).
(ii) Transforms and contour integral methods for differential equations, including the method of steepest descents (lectures 6 to 8; Dr Alex Wray and Dr David Pritchard).
(iii) Further techniques in asymptotics (lectures 9 and 10; Dr David Pritchard).

Prior knowledge: basic ODEs (first-order separable and first- and second-order linear equations); single- and multivariable calculus; Taylor's theorem; linear algebra; contour integration including Cauchy's theorem.

Continuum Mechanics (Semester 1)

(i) Introduction to continuum mechanics, including tensors, kinematics, balance laws and governing equations, constitutive laws (lectures 1 to 4; Dr Penny Davies).
(ii) Fluid dynamics, including Newtonian fluid dynamics and non-Newtonian fluids (lectures 5 to 9; Prof. Nick Hill, Dr Peter Stewart and Prof. Iain Stewart).

Prior knowledge: introductory courses on ODEs, PDEs, vector calculus and basic linear algebra. Prior knowledge of continuum mechanics is not assumed.

Numerical Methods (Semester 2)

(i) Numerical methods for stochastic DEs (lectures 1 and 2; Prof. Des Higham).
(ii) Numerical methods for ODEs, including implicit, explicit and multistep methods (lectures 3 and 4; Prof. Dugald Duncan).
(iii) Numerical methods for PDEs, in particular finite-difference methods (lectures 5 to 8; Prof. Ping Lin and Dr Irene Kyza).
(iv) Numerical linear algebra (lectures 9 and 10; Dr Phil Knight).

Prior knowledge: basic ODEs (in particular first order separable and first- and second-order linear equations); single- and multivariable calculus; Taylor's theorem; and linear algebra. Prior knowledge of Matlab or Octave would be helpful.

Mathematical Biology and Physiology (Semester 2)

(i) Mathematical physiology, including models of air flow and mixing in the human lung, pulse propagation in arteries, the heart and electrophysiology (lectures 1 to 3; Dr Peter Stewart and Dr Radostin Simitev).

(ii) Mathematical biology, including population dynamics, disease models, phenomena on evolutionary time scales, adaptive dynamics of pathogen-host interactions, evolutionary stability and bifurcation theory, host population structure, age-structured models, patterns and waves in Turing models (lectures 4 to 10; Prof. Rachel Norman, Dr Andy Hoyle, Dr Douglas Speirs, Dr Frits Veerman).

Prior knowledge: introductory courses on ODEs, PDEs, vector calculus and basic linear algebra. Prior knowledge of biology is not assumed but some knowledge of fluid dynamics (e.g. the Continuum Mechanics module) may be useful for lectures 1–3.

Delivery. Some lectures will be delivered in "flipped" format, with material to read and exercises to complete before each class. It is essential that you carry out this work in order to be prepared for the class. For "flipped" classes, pre-class material will appear at least a couple of days beforehand, but full notes will appear only *after* each class.

Assessment. Each module will be assessed by two written assignments, which may include both "paper and pencil" and computer work. The provisional deadlines are as follows.

Asymptotic and Analytical Methods

- Assignment 1 (lectures 1–5): to be submitted by 23 November 2018.
- Assignment 2 (lectures 6–10): to be submitted by 6 January 2019.

Continuum Mechanics

- Assignment 1 (lectures 1–4): to be submitted by 23 November 2018.
- Assignment 2 (lectures 5–9): to be submitted by 11 January 2019.

Numerical Methods

- Assignment 1 (lectures 1–4): to be submitted by 1 March 2019.
- Assignment 2 (lectures 5–10): to be submitted by 5 April 2019.

Mathematical Biology and Physiology

- Assignment 1 (lectures 1–3): to be submitted by 22 February 2019.
- Assignment 2 (lectures 4–10): to be submitted by 5 April 2019.

Each assignment will be set at least two weeks before the deadline.

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