Applications of Mathematics

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Overview of the theme

The four modules in this theme are concerned with:

- formulating and analysing **mathematical models**;
- the **tools** we often need to do so.

These modules can be taken independently of each other.

We will look at two major areas of modelling:

- **continuum mechanics**;
- **mathematical biology**.

These will illustrate a range of modelling approaches.

We will also look at two complementary sets of tools:

- **asymptotic** and analytical methods;
- **numerical** methods.

Both allow us to develop approximate solutions in a systematic manner.

Continuum mechanics (semester 1)

Continuum mechanics describes how deformable media (fluids and solids) behave. Applications range from keyhole surgery to climate change.

We will look at topics including:

- **rational continuum mechanics** (how to systematically construct a dynamic model of a deformable medium);
- classical **fluid dynamics** (including lubrication theory, aerofoils and hydrodynamic stability);
- the behaviour of **non-Newtonian** fluids.

Mathematical biology and physiology (semester 2)

Mathematical modelling in the life sciences is a rapidly-evolving research area — with many pitfalls for the inexperienced!

We will look at topics including:

- **mathematical physiology**, including airflow in the lungs, blood flow, and electrophysiology;
- **population modelling**, including epidemiology, evolution, and age-structured models;
- pattern and wave formation in **Turing models**.

Asymptotic and analytical methods (semester 1)

Problems frequently contain a small parameter $\epsilon \ll 1$.

- A small parameter doesn't always have a small effect!
- Even a small effect can't always be determined naïvely.

We will look at topics including:

- modulation and resonance in oscillators (**multiple scales**);
- **matched asymptotics** and boundary layers in BVPs;
- rapidly oscillating solutions (**WKBJ theory**);
- approximation of **integrals** (Watson's Lemma, steepest descents);
- self-similar **intermediate asymptotic** solutions;
- **resummation** techniques for series.

Numerical methods (semester 2)

There are lots of ways to solve ODEs, PDEs etc. numerically.
Unfortunately, the obvious ones aren't always the best:

- they may be horribly inefficient;
- their output may be pretty but unreliable.

We will look at topics including:

- explicit and implicit **methods for ODEs**;
- an introduction to **stochastic DEs** and their solution;
- finite-difference and finite-element **methods for PDEs**;
- efficient numerical **linear algebra**.

Prior knowledge, delivery and assessment

We assume you're familiar with standard undergraduate material, e.g.:

- calculus, ODEs and PDEs;
- linear algebra;
- complex variables.

See the handout for further details.

Lectures: Wed 1300–1500 (Methods); Thu 1530–1730 (Models).
Lectures may be "flipped", with exercises to complete beforehand — including before the first lecture!

Assessment will be based on two coursework assignments per module.

- Each assignment covers three to seven lectures.
- Assignments may include "paper and pencil" and computer work.
- You'll have at least two weeks to tackle each assignment.